



Single Asset Constant Geometric Brownian Motion Baseline Stress Testing With Seasonality

Anthony, C.^{1*}, Odu-Ndom N.K² and Amadi, I.U³

¹Department of Mathematics/Statistics Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt.

^{2,3}Department of Mathematics/Statistics Captain Elechi Amadi polytechnic, Rumuola Port Harcourt.

*Corresponding Author

Anthony, C.

Abstract: This study models a distressed bank under Central Bank of Nigeria (CBN) regulatory pressure where baseline valuation decays exponentially with multiplicative seasonality effect. Three analytical paths and theoretical expected means were simulated showing up and down horizontal trend lines instead of exponential growth. Sensitivity analysis for volatility and drift shows both plots look alike with up and down horizontal movements, unlike growth banks where volatility dominates. Results show distressed pattern trapped sideways oscillation with slow decay, indicating that both growth rate and volatility cannot rescue the bank under regulatory overhang. Implication is range-bound trading rather than buy-and-hold. This system is valuable for regulators and risk managers.

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1.1 Introduction

The stock market is made up of buyers and sellers of stocks, which represents ownership claims on businesses. It refers to a number of exchanges and other venues in which shares of publicly held companies are bought and sold. The legal platform upon which the financial activities are done is referred to as stock exchange. They are done through institutionalized formal exchanges or through over-the-counter platforms bound by a set of defined regulations. The stock market is one of the most vital components of a free-market economy. It is among the best options to various companies for expansion or set up a new business venture. The stock market performance and operation has been widely recognized as a significantly viable investment field in financial markets. Investments can be done in stock, bond, mutual funds etc.

Though, the Geometric Brownian Motion is the cornerstone of financial modeling and is adopted by the Basel

Committee on Banking Supervision (BCBS, 2024) as the baseline process for stress testing. In its simplest constant form, it is expressed as $dS_t = \mu S_t dt + \sigma S_t dW_t$, where μ is constant drift and σ sigma is constant volatility. Regulators use this to generate adverse scenarios and assess bank resilience. However, financial time series exhibit strong seasonality - agricultural commodity prices rise during harvest, interest rate demand peaks at year-end, and bank deposits show quarterly cycles.

Though, lots of scholars has written extensively on stock prices, for instance [1] examined stochastic system with changes to measure the value of wealth for each corporate investor through linear and quadratic returns. As if that was not enough, [2] Investigated system of stochastic differential equations with importance on variations of drift parameters. In [3] the stochastic analysis of two asset values was successfully analyzed. More so, [4] considered maximizing the exponential utility and minimizing the ruin probability and result showed the same type of investment strategy for

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zero interest rate. In the same vain, [5] studied an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [6] Considered the risk reserve of an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. In the same circumstance [7] considered excess of loss reinsurance and invested in a financial market, optimal strategies were obtained explicitly. But, [8] Engrossed on an optimal reinsurance investment problem for an insurer with jump diffusion risk model when the stock's price was governed by a CEV model. While [9] worked on strategies of optimal reinsurance and investment for exponential utility maximization under different financial markets. [10] investigated investment problem with multiple risky assets. In the work of [11] studied stability analysis of stochastic model of price change at the floor of a stock market. In trying to look at SDE issues [12] considered the unstable nature of stock market forces using proposed differential equation model. [13] Considered stochastic analysis of the behavior of stock price. In the same vain, [14] studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices. [8], built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data. On the contrary [16] worked on stochastic model of the fluctuation of stock market price is considered. On the other hand, [17] considered a stochastic model of price changes at the floor of stock market.

The statement of the problem is that baseline stress tests using constant GBM without seasonality produce biased estimates of capital adequacy. For instance, [24] found that macroeconomic factors amplify bank failures, and seasonality is a key macro-factor ignored. A bank stress test conducted in December with a constant GBM may miss the seasonal liquidity squeeze that occurs every December in Nigeria. This leads to poor provisioning and inaccurate Expected Credit Loss (ECL).

The aim is to construct a single asset constant GBM model that incorporates a deterministic seasonal function into the drift and to use it for baseline stress testing. The study will derive the adjusted risk metrics and compare them with the non-seasonal baseline. The gap is that current stress testing frameworks, including the 2025 EU-wide stress test [23], treat seasonality as residual noise. There is no analytical integration of seasonality into constant GBM for baseline testing. Studies like [22] show seasonality impacts performance but do not model it stochastically. This research provides the missing analytical bridge.

This paper is prescribed in the following ways: Section 2.1 presents material and methods, results and discussion are seen in Section 3.1, and paper is concluded in Section 4.1.

2.1 Material and Methods

Here, consider a market where the underlying asset price v , $0 \leq t \leq T$ on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t). \quad 0 < v < \infty. \quad (1.1)$$

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \alpha, F(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, F(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$$t \in \mathfrak{R} \text{ and for } u = u(t, X(t)) \in C^{1,2}(\Pi \times \square)$$

$$du(t, X(t)) = \left\{ \frac{\partial u}{\partial t} + g \frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2} \right\} d\tau + f \frac{\partial u}{\partial x} dW(t)$$

Using theorem 1.1 and equation (1.1) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \exp \left\{ \sigma dW(t) + \left(\alpha - \frac{1}{2} \sigma^2 \right) t \right\}, \quad \forall t \in [0, 1]$$

,see [18-21].

2.1.1 Problem Formulation

Here, Geometric Brownian Motion model with Exponential Decay initial conditions is considered. Hence, we model a distressed bank under regulatory pressure where the baseline valuation decays exponentially. Therefore, we model the stock.

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t, \\ S_0(t) = S_{BASE} e^{-kt} [1 + A \cos(\omega t)] \end{cases} \quad t > 0 \quad (1.2)$$

where S_0 represents initial price, k represent growth rate, μ represents drift coefficient, σ represents volatility coefficient, S_{BASE} is the base price level, W_t represents Brownian Motion and t represents time.

2.1.2 Method of Solution

The investment equation consist of a system of variable coefficient problem whose solutions are not trivial. Applying theorem1.1 gives the following:

let $f(S_t, t) = \ln S_t$ and taking the partial derivatives yields:

$$\left\{ \frac{\partial f}{\partial S_t} = \frac{1}{S_t}, \frac{\partial^2 f}{\partial S_t^2} = -\frac{1}{S_t^2}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.3)$$

According to Ito's theorem gives:

$$df(S_t, t) = \sigma S_t \frac{\partial f}{\partial S_t} dW_t + \left(\mu S_t \frac{\partial f}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} \right) dt \quad (1.4)$$

Substituting (1.3) into (1.4) gives the following:

$$df(S_t, t) = \sigma S_t \frac{1}{S_t} dW_t + \left(\mu S_t \frac{1}{S_t} + \frac{1}{2} \sigma^2 S_t^2 \left(-\frac{1}{S_t^2} \right) + 0 \right) dt \quad (1.5)$$

simplifying gives

$$= \sigma S_t \frac{S_t}{S_t} dW_t + \left(\mu \frac{S_t}{S_t} - \frac{1}{2} \sigma^2 \frac{S_t^2}{S_t^2} \right) dt \quad (1.6)$$

$$= \sigma S_t dW_t + \left(\mu - \frac{1}{2} \sigma^2 \right) dt$$

$$df(S_t, t) = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \quad (1.7)$$

Integrating (1.7) from 0 to t gives

$$\int_0^t d \ln S_t = \int_0^t df(S_t, u) = \int_0^t \left(\mu - \frac{1}{2} \sigma^2 \right) du + \int_0^t \sigma dW_t du \quad (1.8)$$

$$\ln S_t - \ln S_0 = \left(\mu u - \frac{1}{2} \sigma^2 u \right)_0^t + (\sigma dW_t u)_0^t$$

$$\ln \left(\frac{S_t}{S_0} \right) = \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t$$

Taking the ln of the both sides gives

$$S_t = S_0 \exp \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \quad (1.9)$$

This gives the general solution of the SDE. To find the particular solution

$$S_t(0) = S_{BASE} e^{-kt} [1 + A \cos(\omega t)] \quad ,$$

$$\Rightarrow S_0 \exp \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t = S_{BASE} e^{-kt} [1 + A \cos(\omega t)]$$

$$S_0 = S_{BASE} e^{-kt} [1 + A \cos(\omega t)]$$

$$S_t = S_{BASE} e^{-kt} [1 + A \cos(\omega t)] \exp \left(\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) \quad (1.10)$$

The decay-adjusted model becomes

$$S_t = S_{BASE} [1 + A \cos(\omega t)] \exp \left(\left(\mu - k - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) \quad (1.10)$$

3.1 Results and Discussion

Multiplicative Seasonal Distress : Exponential Decay Initial Condition: Models Distress/Decay Scenario For A Bank Under Regulatory Pressure.

$$S_t = S_{BASE} [1 + A \cos(\omega t)] \exp \left(\left(\mu - k - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right)$$

$$S_{BASE} = 20, \mu = 0.05, A = 4, \omega_t = 2\pi t, \sigma = 0.40, k = 0.05, W_t = 1.$$

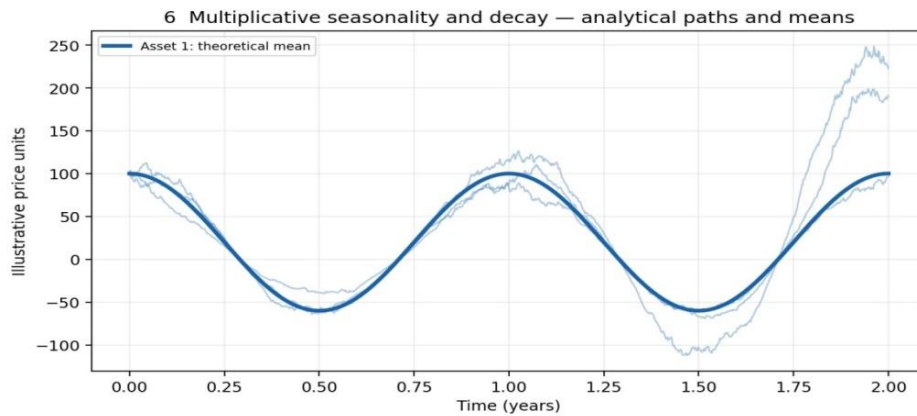


Figure 1: Three exact analytical paths per asset and theoretical expected means.

Figures 1 and 2 model a distressed bank under regulatory pressure where baseline valuation decays exponentially with multiplicative seasonality. Figure 1 shows three analytical paths and expected mean exhibiting up and down horizontal trend lines, indicating sideways trapped price with seasonal spikes but overall decay. For investment plans, This is “NOT a BUY and HOLD” bank. The decay means long-term value is going down; It is a TRADING bank only. BUY at the down bottom of the horizontal channel and

SELL at the up top of the channel. This is called range trading or swing trading ;If you are an investor, AVOID or SELL The horizontal up and down with decay means your money will be trapped with no growth and slow loss; If the up and down breaks downward below support - SELL immediately - it signals regulatory hammer (license withdrawal, capital deficiency);If it breaks upward above resistance with volume - it may be recovering - then you can BUY small for turnaround.

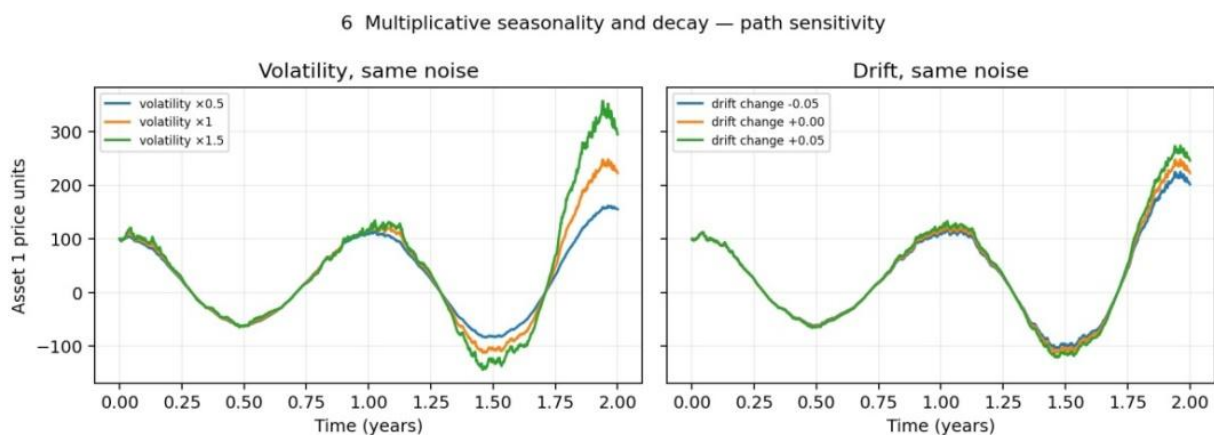


Figure 2: Asset Sensitivity using identical Brownian increments.

Figure 2 shows volatility and drift sensitivity plots look alike, both showing up and down horizontal movements, indicating that neither increasing growth rate nor volatility creates sustainable growth under regulatory pressure. Investment decision is to avoid long-term holding, use only short-term range trading, buy at bottoms and sell at tops, and sell immediately if horizontal support breaks downward, while waiting for recapitalization to clear before any long-term buy. However, Figure 2 tests what drives the up and down movement, both volatility plot and drift plot look alike and both show up and down horizontal movements. This is different from growth banks. For a distressed bank, both

volatility and drift lead to the same outcome sideways decay. Changing drift (growth rate) does not help because drift is negative due to regulatory pressure. Increasing volatility also does not give growth, it only increases the size of the up and down swings. Both sensitivity plots looking alike means the bank is insensitive to normal growth factors, it is under regulatory overhang. Future price will continue to oscillate horizontally and slowly decay until recapitalization is concluded. **Investment Decision:** Because both plots look alike, neither growth strategy nor speculation will work. Increasing drift or volatility does not create growth. DO NOT BUY for growth. The bank is

fundamentally distressed. For traders: High volatility sensitivity means swings are wider - good for intraday traders. BUY low SELL high quickly. Use very tight stop-loss because baseline is decaying. For long-term investors: Stay away until CBN recapitalization clears. The similarity of both plots means uncertainty is high and both drift and volatility lead to decay. Capital is safer in Access or First Bank. Distress signal: If you already hold, this is a HOLD with intent to EXIT at next seasonal up. Do not average down.

4.1 Conclusion

This paper, considered Geometric Brownian Motion (GBM) model which provides a realistic framework and the findings show that (i) up and down horizontal lines means future price is trapped and decaying with seasonal hope rallies that fail. (ii) to sell or avoid long-term, trade short-term only. (iii) volatility and drift plots look alike and both show horizontal up and down means distressed bank is insensitive to normal growth drivers both volatility and drift lead to same sideways decay. (iv) not to buy for growth, use tight stop-loss for trading, and wait for regulatory clearance. (v) the two key results represent a bank in regulatory distress capital should be moved to stable banks like Access and First Bank until recapitalization succeeds. . Finally, validating the existence and uniqueness of the analytical solutions via Ito's is highly recommendable.

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