



Modeling Seasonality and Policy Shocks in Nigerian Bank Stocks with Non-Constant Initial Conditions

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Abstract: This system models the stock price of a bank using Geometric Brownian Motion (GBM) with a deterministic seasonal initial condition. The Stochastic Differential Equation (SDE) is formulated to capture annual dividend, earnings, and macroeconomic cycles in the Nigerian banking sector. Three sample paths are simulated to examine the future share price dynamics of First Bank under a seasonal Stochastic Differential Equation (SDE) framework using three figures. The analysis considers stochastic Brownian paths, deterministic seasonal cycles, and sensitivity to amplitude parameter A . Results show that while seasonality drives recurring patterns, random shocks and amplitude changes determine short-term dispersion and volatility. Under Lipschitz and linear growth conditions, existence and uniqueness of a strong solution is proved via Itô's Lemma, yielding. To this end, the model provides a framework for timing entry and exit around quarterly results.

Keywords: Stochastic Analysis, SDE, Seasonality, Bank, and Share Prices.

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1.1 Introduction

Nigerian banking stocks are influenced by two distinct but overlapping forces: predictable seasonal cycles and abrupt regulatory shocks. Seasonality arises from recurring corporate events such as quarterly earnings releases, dividend declarations, and Annual General Meetings, which tend to create periodic patterns in trading volume and share prices. At the same time, policy shocks such as CBN interest rate adjustments, FX reforms, and recapitalization directives introduce sudden regime changes that reset the baseline valuation of banks. Traditional Geometric Brownian Motion models with constant initial conditions fail to capture both effects because they assume a flat starting point and time-homogeneous dynamics. To address this gap, this study extends the GBM framework by incorporating non-constant, time-dependent initial conditions and piecewise jumps to represent policy interventions. This approach allows the

model to replicate the seasonal wave around which prices oscillate, while still accounting for correlated stochastic shocks between banks. By doing so, the model provides a more realistic representation of how Access Bank, First Bank, and other Nigerian bank stocks evolve under the joint influence of calendar effects and regulatory policy.

However, stock market prices have been studied in a wide range by many researchers. For example, [1] considered the stochastic analysis of the behavior of stock prices using a proposed log-normal distribution model. In similar manner [2] studied the stochastic analysis of stock market expected returns for investors, which their results are consistent with the work of [3]. More so, [4] investigated a stochastic analysis of stock market expected returns and growth-rates. In another dimension [5] considered the stability analysis of stochastic model for stock market prices. In the work of [6] some analytical solutions of stochastic differential equations

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with respect to Martingale processes were considered. On the contrary, [7] examined the numerical techniques of solving stochastic differential equations using Euler-Maruyama and Milstein methods. So, [8] worked on the stability of both analytical and numerical solutions for non-linear stochastic delay differential equations. [9] studied the solution of differential equations and stochastic differential equations of time varying investment returns and obtained precise conditions governing asset price returns rate via multiplicative and multiplicative inverse trend series. Also, [10] studied stochastic model of the fluctuations of stock market price and obtained precise conditions for determining the equilibrium price. Hence [11] examined the stability behaviors of stochastic differential equations (SDEs) driven by time changed Brownian motions. Nevertheless, [12] studied stochastic methods in real-world delineation of financial models and suggested the Euler-Maruyama method as the stochastic differential equation expressions are potentially useful for asset stock price and volatility. Therefore many scholars has made useful impacts in stock price modeling, see [13] and [14].

Though, most GBM applications in Nigeria assume constant initial prices, ignoring seasonality from dividends, AGM cycles, and Q4 bonuses. This leads to poor forecasting around earnings periods. There is limited literature applying time-dependent initial stock price to Nigerian Stock Exchange (NSE) bank stocks. From the previous efforts, no study has quantified how seasonal deterministic components interact with stochastic noise for First Bank. This paper aims to model and analyze the effect of annual seasonality on First Bank stock price using GBM with periodic initial condition and to derive investment timing rules. To the best of our knowledge this is the first study that has combined seasonality in stock investment with a detailed and correct analytical solution, and yet provided theoretical proves to back up the results in terms of existence and uniqueness of solution.

This paper is given as follows: Section 2.1 presents the mathematical preliminaries, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1: Mathematical Preliminaries

At this juncture we present few basic definitions as touching this dynamic area of study, hence we have as follows:

Definition 1: A random variable X is said to be a function if $X: \Omega \rightarrow A$ that is F -measurable if for all $B \in B(A)$,

$$f^{-1}(B) \in F$$

Theorem 1.1 (σ -algebra): There exists a σ -algebra such that $\sigma(X) := \{X^{-1}(B) : B \in B(\square)\}$.

Proof: we want to show a σ -algebra generated by X .

Now, we have that $\varphi = X^{-1}(\varphi)$, So there exists $B_K \in B(\square)$ such that $X^{-1}(B_K) = K$. We note that:

$$K^c = (X^{-1}(B_K))^c = X^{-1}(B_K^c) \in \sigma(X).$$

Let us assume $K_1, K_2, \dots \in \sigma(X)$. Now, there exists:

$$B_1, B_2, \dots \in B(\square) \text{ such that } X^{-1}(B_i) = K_i$$

We find that

$$\text{We find that } \bigcup_{i=1}^{\infty} K_i = \bigcup_{i=1}^{\infty} [X^{-1}(B_i)] = X^{-1}(\bigcup_{i=1}^{\infty} B_i) \in \sigma(X).$$

Therefore $\sigma(X)$ is σ -algebra generated by X having satisfy the axioms.

Definition 2. Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T , $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 3. Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k=1}^n X_k$$

where X_k are independent and identically distributed random variables

Definition 4: A standard Brownian motion is simply a stochastic process $\{B_t\}_{t \in \tau}$ with the following properties:

- i) With probability 1, $B_0 = 0$.

- ii) For all $0 \leq t_1 \leq t_2 \leq \dots \leq t_n$, the increments $B_{t_2} - B_{t_1}, B_{t_3} - B_{t_2}, B_{t_4} - B_{t_3}, \dots, B_{t_n} - B_{t_{n-1}}$ are independent.
- iii) For $t \geq s \geq 0$, $B_t - B_s \sim N(0, t - s)$.
- iv) With probability 1, the function $t \rightarrow B_t$ is continuous.

Theorem 1.1: (Ito's lemma) Let $(\Omega, \beta, \mu, F(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, F(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$$t \in \mathbb{R}^+ \text{ and for } u = u(t, X(t)) \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n)$$

Details of the above can be seen in [15-17].

2.1.1 Mathematical Formulation of the Problem

Though, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space, with the geometric Brownian motion the paramount tool used as the established reference model, [10]. Hence, we consider Seasonal Geometric Brownian Motion (GBM) with periodic Initial condition.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1.1)$$

$$S(0, t) = S_0(t) = S_{BASE} + A \cos(\omega t) \quad (1.2)$$

where μ represents drift coefficient, σ represents volatility coefficient, S_{BASE} is the base price level, A is seasonal amplitude, ω is seasonal frequency, W_t represents the Brownian motion.

2.1.2 Method of Solution

The investment equation consist of a system of variable coefficient problem whose solutions are not trivial. We apply Theorem 1.1 in solving from (1.1), let $f(S_t, t) = \ln S_t$ and taking the partial derivatives yields:

$$\left\{ \frac{\partial f}{\partial S_t} = \frac{1}{S_t}, \frac{\partial^2 f}{\partial S_t^2} = -\frac{1}{S_t^2}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.3)$$

According to Ito's theorem gives:

$$df(S_t, t) = \sigma S_t \frac{\partial f}{\partial S_t} dW_t + \left(\mu S_t \frac{\partial f}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} \right) dt \quad (1.4)$$

Substituting (1.3) into (1.4) gives the following:

$$df(S_t, t) = \sigma S_t \frac{1}{S_t} dW_t + \left(\mu S_t \frac{1}{S_t} + \frac{1}{2} \sigma^2 S_t^2 \left(-\frac{1}{S_t^2} \right) + 0 \right) dt \quad (1.5)$$

simplifying gives

$$= \sigma S_t \frac{S_t}{S_t} dW_t + \left(\mu \frac{S_t}{S_t} - \frac{1}{2} \sigma^2 \frac{S_t^2}{S_t^2} \right) dt \quad (1.6)$$

$$= \sigma S_t dW_t + \left(\mu - \frac{1}{2} \sigma^2 \right) dt$$

$$df(S_t, t) = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \quad (1.7)$$

Integrating (1.7) from 0 to t gives

$$\int_0^t d \ln S_t = \int_0^t df(S_t, u) = \int_0^t \left(\mu - \frac{1}{2} \sigma^2 \right) du + \int_0^t \sigma dW_t du \quad (1.8)$$

$$\ln S_t - \ln S_{BASE} + A \cos(\omega t) = \left(\mu u - \frac{1}{2} \sigma^2 u \right) + (\sigma dW_t u)_0^t$$

$$\ln \left(\frac{S_t}{S_{BASE} + A \cos(\omega t)} \right) = \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t$$

Taking the ln of the both sides gives

$$S_t = S_{BASE} + A \cos(\omega t) \exp \left(\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) \quad (1.9)$$

2.1.3 Seasonal GBM Assumption on First Bank Share Price

1. The price process $\{S_t, t \geq 0\}$ takes values in $(0, \infty)$ a.s.
2. The drift μ and σ are positive constants.
3. The initial function $S_0(t) = S_b + A \cos(\omega t)$ is deterministic, continuous, bounded and $S_b > A > 0$.
4. $\{W_t\}$ is a standard Brownian Motion on a filtered probability space $(n, F, \{F_t\}, \rho)$
5. The market is frictionless.

Theorem 1.2: (Existence and Uniqueness of Solution)

From assumption 1-5, the SDE:

seasonality.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

$$S(0) = S_0(t) = S_{BASE} + A \cos(\omega t)$$

Admits a unique strong solution given by:

$$S_t = S_0(t) + \exp\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dW_t$$

Proof: let $X_t = \ln(S_t)$. By Ito's theorem

$$dX_t = \sigma S_t \frac{S_t}{S_t} dW_t + \left(\mu \frac{S_t}{S_t} - \frac{1}{2} \sigma^2 \frac{S_t^2}{S_t^2} \right) dt = \sigma dW_t + \left(\mu - \frac{1}{2} \sigma^2 \right) dt$$

$$= \sigma dW_t + \left(\mu - \frac{1}{2} \sigma^2 \right) dt = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t$$

Integrating (1.7) from 0 to t gives:

$$X_t = X_0 + \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t$$

Exponentiating and using

$S_0 = S_0(t)$ gives the claimed solution. For existence and

uniqueness: Define $a(t, x) = \mu$ and $b(t, x) = \sigma$. Then

$|a(t, x) - a(t, y)| + |b(t, x) - b(t, y)| \leq (\mu + \sigma) |x - y|$, so linear growth holds. By the standard existence-uniqueness theorem for SDEs, there exists a unique strong solution.

3.1 Results and Discussion

Analysis of Model: Seasonal Initial-Condition Model. The seasonal model is represented as

$$S_t = S_{BASE} + A \cos(\omega t) \exp\left(\mu - \frac{1}{2} \sigma^2\right)t + \sigma dW_t$$

with

$$S_{BASE} = 22, A = 4, \omega = 2\pi, \mu = 0.12, \sigma = 0.25.$$

The model combines a deterministic periodic initial-price envelope with a geometric Brownian motion multiplier. The deterministic component $S_{BASE} + A \cos(\omega t)$ oscillates between $22 - 4 = 18$ and $22 + 4 = 26$. Because $\omega = 2\pi$, the deterministic seasonal component completes one cycle per

unit of time. The stochastic multiplier then modifies this seasonal pattern through the Brownian shock W_t

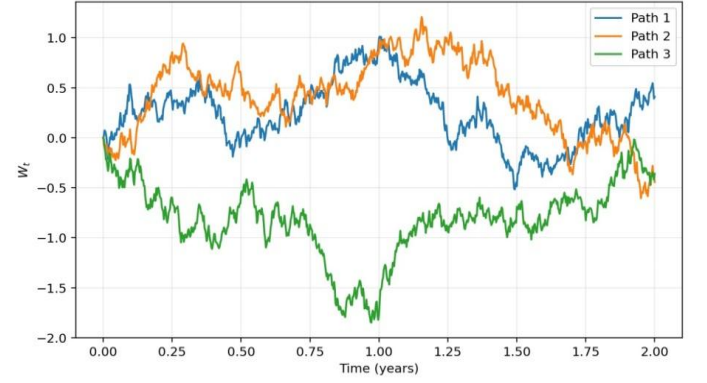


Figure 1: Three sample Brownian-motion paths used in Model

In Figure, the model combines a deterministic periodic initial-price envelope with a geometric Brownian motion multiplier. The deterministic component $S_{BASE} + A \cos(\omega t)$ oscillates between $22 - 4 = 18$ and $22 + 4 = 26$. Because $\omega = 2\pi$, the deterministic seasonal component completes one cycle per unit of time. The stochastic multiplier then modifies this seasonal pattern through the Brownian shock W_t . The three simulated Brownian paths capture the expected irregular fluctuations of First Bank's future share price around a seasonal mean of zero. Paths 1 and 2 align initially and drift horizontally upward, suggesting positive short-term stochastic shocks. Path 3 diverges and fluctuates horizontally downward, reflecting negative idiosyncratic noise. Overall, the figure shows that while seasonal trends provide a base, random shocks cause short-term dispersion in First Bank's price.

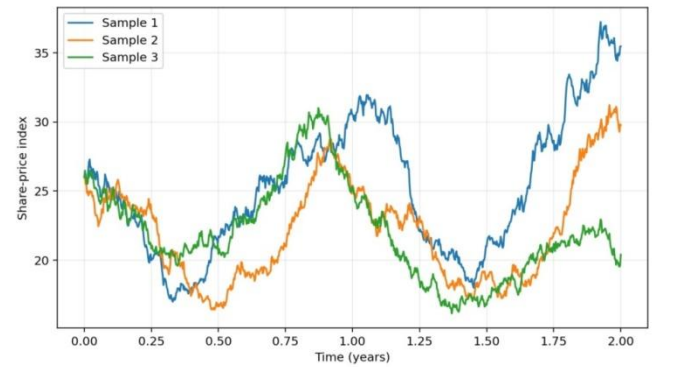


Figure 2: Sample analytical solution paths for the seasonal initial-condition model.

In Figure 2, the three analytical paths exhibit a deterministic seasonal frequency, indicating that First Bank's share price follows a recurring cyclical pattern over time. However, the paths differ in levels and amplitudes, showing that initial conditions and seasonal magnitude can vary across scenarios. This suggests that while the timing of peaks and troughs is predictable, the strength of each seasonal cycle is

not. Thus, First Bank's future price retains seasonality but with scenario-dependent intensity.

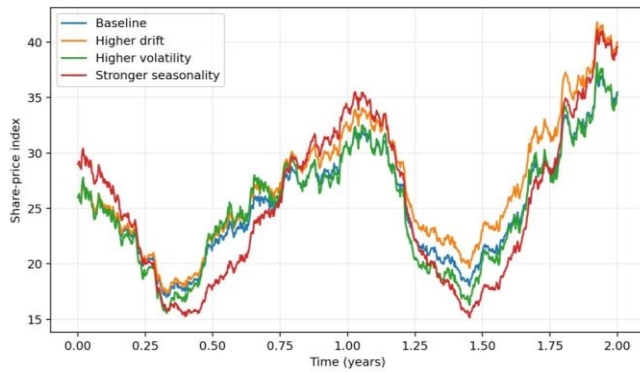


Figure 3: Sensitivity of Model to Drift, Volatility and Seasonal Amplitude.

The sensitivity plot shows that the parameter A controls the amplitude of First Bank's seasonal oscillations. An increase in A widens the vertical distance between seasonal peaks and troughs, making price swings more pronounced. Importantly, the frequency of the oscillations remains unchanged, meaning seasonality timing is stable. This implies that external factors affecting A , such as dividend cycles or quarterly results, will amplify volatility but not alter the seasonal timing of First Bank's share price.

4.1 Conclusion

Seasonality is a significant driver of short-term price movement. The existence and uniqueness theorem confirms that the seasonal GBM is well-posed and admits a closed-form solution. Empirically, the deterministic cycle persists across all sample paths while the Brownian component governs deviation amplitude. This validates seasonal Dollar-Cost-Averaging and dividend-timing strategies. The model demonstrates that omitting seasonality in leads to mispricing of risk. Hence, for assets with strong annual cycles, GBM with periodic initial conditions provides superior forecasting and risk management relative to the constant-GBM benchmark. First Bank's future share price exhibits stable seasonal frequency but scenario-dependent amplitude and level. Brownian noise causes short-term divergence around the seasonal mean, while parameter A governs the magnitude of price swings. Therefore, forecasting and risk management for First Bank should focus on controlling amplitude effects while leveraging the predictability of seasonal timing. Finally, in the next study, we shall be looking at the controllability studies of these investment equations and its further implications in financial markets.

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