



Kelvin Effect and Hygroscopic Growth of Mineral Nucleation Component (MINN) under Varying Relative Humidity: A Koehler Theory Approach

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Abstract: This study presents the results of the analysis of the influence of the Kelvin curvature and hygroscopic growth on the optical and equilibrium properties of desert aerosols components under varying relative humidity (RH). Microphysical properties (volume mix ratios, dry mode radii and wet mode radii) of desert aerosols were extracted from the OPAC 4.0 (Optical Properties of Aerosols and Clouds) and analyzed to quantify the influence of Kelvin Curvature (Kelvin effect) and water activities on equilibrium relative humidities, effective radius and effective hygroscopic growth. Data were extracted over the spectral range of (0.4 to 0.8 μm) and at eight relative humidities of (0%, 50%, 70%, 80%, 90%, 95%, 98%, and 99%). The concentrations of Mineral Nucleation Non-Spherical Mode (MINN) were varied to obtain five different models. Koehler theory was applied to determine the effective hygroscopic growth factor, effective radius, Kelvin effect, water activity, equilibrium vapour pressure, and first-order Kelvin corrections, while the empirical γ -model, k-model, and three-parameter growth model were used to evaluate hygroscopic growth behaviour. The results showed that the effective hygroscopic growth factor increased with increasing relative humidity of Mineral Nucleation Non-Spherical Mode (MINN) concentration, whereas the effective radius and Kelvin effect decreased. Water activity approached unity, while equilibrium vapour pressure increased rapidly near saturation. Kelvin effect corrections of the effective hygroscopic growth factor and effective radius remained negligible below 90% relative humidity but increased between 95% and 99% relative humidity. The regression analysis produced R^2 values of 0.99905 to 0.99929, which show data fitted equation very well, with A ranging from -0.0181 to -0.0137, and B from 0.0208 to 0.0442 across models 1 to 5. The k- model gave $R^2 = 0.99912$ to 0.99913, while the three-parameter model gave $R^2 = 0.99925$ to 0.99928. Model 1 consistently showed the largest corrections value and Model 5 the smallest correction. The study demonstrated that increasing mineral aerosol concentration enhanced water uptake while reducing the relative influence of Kelvin curvature.

Keywords: Desert aerosols, Effective hygroscopic growth, Effective radii, Kelvin effect, Relative Humidity.

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1. INTRODUCTION

Aerosols are liquid or solid particles suspended in the air. They have a direct effect on climate because they scatter and absorb solar and infrared radiation in the atmosphere and greatly degrade visibility (Zieger *et al.*, 2013). Aerosols also alter warm, ice and cloud formation processes by increasing droplet number concentrations and ice particle concentrations (Jacob *et al.*, 1986).

Desert atmospheric aerosols, particularly mineral dust, are important components of the Earth's atmosphere. They are emitted mainly from arid and semi-arid regions and can be transported over long distances. During transport, dust particles interact with radiation, clouds and atmospheric water, thereby affecting climate, air quality and visibility (Kok *et al.*, 2023). The interaction of dust particles with atmospheric water is particularly important because it changes their size, effective radius and optical properties and can influence their ability to act as cloud condensation nuclei CCN.

The amount of water taken up by mineral dust depends on relative humidity, particle size, mineral composition and surface properties. Mineral dusts absorb and scatter radiation while on the other hand they act as cloud condensation nuclei (CCN) and ice nuclei (IN) leading to cloud formation and growth as well as influencing albedo, persistence, and other cloud properties (Camon *et al.*, 2025). The aerosol particles uptake moisture when the relative humidity increases, as such are more soaked with water from the surrounding humid air and swell (Lang *et al.*, 1980). Previous studies have also shown that water uptake differs among mineral surfaces, demonstrating the importance of mineral composition in controlling hygroscopic growth Li and Bourg (2024). However, the studies on the combined effects of increasing mineral aerosol concentration, relative humidity, water activity, equilibrium vapour pressure and Kelvin Effect corrections on the hygroscopic growth of the components of desert aerosols (Mineral Nucleation Non- spherical) Mode MINN are still limited.

Koehler theory provides a useful framework for describing aerosol and water equilibrium by considering both water activity and the Kelvin curvature effect. The Kelvin effect increases the equilibrium vapour pressure over a curved particle surface and may become more important as particles approach saturation. The Koehler approach and other hygroscopic growth parameterizations have been widely used to describe aerosol water uptake, but mineral dust remains difficult to represent because of its complex composition and surface properties. Therefore, a better understanding of the relationship between mineral composition, relative humidity and Kelvin effects is needed.

In this paper the volume mix ratios, dry mode radii and wet mode radii of desert aerosols were extracted from OPAC (4.0) at the spectral wavelength of 0.4 to 0.8 μ m, and at relative humidities of 0%, 50%, 70%, 80%, 90%, 95%, 98% and 99%. From the four components of desert aerosols (WASO, MINN, MIAN, MICN), the Mineral Nucleation Non-Spherical Mode MINN was varied. The parameters were analyzed using excel, SPSS, Origin and Koehler theory and examine five MINN compositions at RH values from 0% to 99%. In addition to effective radius and hygroscopic growth, the study evaluates the Kelvin contribution and its first-order and fractional corrections using the γ -model, k-model and three-parameter model.

2. REVIEW OF PREVIOUS WORK

There are many literatures related to the influence of the Kelvin curvature and hygroscopic growth on the atmospheric aerosols' components under varying relative humidity. This section describes the relevant studies that have been carried out by researchers globally.

Studies applying Koehler-based approaches to mineral and atmospheric aerosols, including Tijjani *et al.* (2015) and Rastak *et al.* (2017), provide further support for relating hygroscopic growth to aerosol composition and humidity. However, the combined behaviour of the Mineral Nucleation Mode (MINN) of desert aerosols under systematically increasing MINN concentration, together with water activity, equilibrium vapour pressure and first-order Kelvin corrections over a wide RH range, is not sufficiently documented in the present context.

Research on mineral and desert aerosols indicates that mineral particles cannot always be treated as completely non-hygroscopic. Mineral dusts absorb and scatter radiation while on the other hand they act as cloud condensation nuclei (CCN) and ice nuclei (IN) leading to cloud formation and growth as well as influencing albedo, persistence, and other cloud properties (Camon *et al.*, 2025). More recent work has continued to emphasize the importance of mineral dust in atmospheric processes and climate, including its effects during atmospheric transport (Kok *et al.*, 2023) and the role of mineral surfaces in water interactions (Li & Bourg, 2024).

Several studies have developed parameterizations for representing aerosol hygroscopic growth. Topping *et al.* (2005) demonstrated the importance of particle curvature in describing hygroscopic growth using a curved-surface formulation. Petters and Kreidenweis (2007) introduced the single-parameter k representation, which provides a convenient way of describing aerosol water uptake and Cloud Condensation Nuclei activity. Wex *et al.* (2008) also examined the role of hygroscopic growth and particle properties in aerosol water interactions. These studies show

that both particle composition and humidity are important in determining aerosol growth.

Koehler theory has long provided the physical basis for describing the equilibrium between an aerosol particle and surrounding water vapour. Koehler (1936) showed that equilibrium is controlled by the combined effects of water activity and the curvature of the particle surface. Pruppacher and Klett (1997) and Seinfeld and Pandis (2006) further established the importance of solution and curvature effects in cloud and aerosol microphysics. These studies provide the theoretical foundation for examining hygroscopic growth and Kelvin curvature in aerosol particles.

The study of atmospheric aerosols has established that aerosol particles influence radiation, cloud formation, visibility and climate. Hess *et al.* (1998) developed OPAC as a widely used database and software package for describing the microphysical and optical properties of aerosol and cloud components. OPAC therefore provides a useful basis for investigating the behaviour of different aerosol components, including mineral particles, under different relative humidity conditions.

3. Theoretical Framework

According to Koehler theory” (Koehler, 1936, Pruppacher & Klett, 1997, Seinfeld & Pandis, 2006), the condition necessary for an aqueous solution droplet to be in equilibrium with water vapor in the surrounding gas phase can be expressed as follows (Kreidenweis *et al.*, 2005, Gysel *et al.*, 2002, Koehler *et al.*, 2006).

$$S = a_w K_e \quad (1)$$

where a_w is the activity of water in the aqueous solution or Raoult term, and K_e is the Kelvin effect, which describes the enhancement of the equilibrium water vapor pressure due to surface curvature (Ruehl *et al.*, 2010).

But generally atmospheric aerosols usually comprised mixtures of soluble and insoluble components, therefore the information on the hygroscopicity modes was merged into an “over-all” or “bulk” or “effective” hygroscopic growth factor of the mixture, $g_{eff}(S)$, representative for the entire aerosols’ particles population as:

$$g_{eff}(S) = (\sum_k x_k g_k^3(S))^{1/3} \quad (2)$$

The effective or volume equivalent radius of the mixture was determined using the relation

$$r_{eff}(S) = (\sum_k x_k r_k^3)^{1/3} \quad (3)$$

where the summation is performed over all compounds present in the particles and x_k represent their respective volume fractions “using the Zdanovskii-Stokes-Robinson

(ZSR) relation (Sjogren *et al.*, 2007, Stokes & Robinson, 1966, Meyer *et al.*, 2009, Stock *et al.*, 2011)”.

Therefore, for atmospheric aerosols, equation (1) can be written to represents the property of the bulk components using equations (2) and (3) as:

$$\ln S = \frac{A}{r_{eff}(S)} + \frac{B}{1 - g_{eff}^3(S)} \quad (4)$$

using multiple regression analysis with SPSS 16.0 for windows, the constants A and B were determined.

The first term on right hand side of Equation (4) becomes:

$$K_e = \exp\left(g_\sigma(r_{eff}(S))\right) \quad (5)$$

where a characteristic length for the effect of surface tension on the mixture or the effective Kelvin radius $r_K = \frac{2v_w \sigma_{eff}}{RT} =$

$$A \text{ and } g_\sigma(r_{eff}(S)) = \frac{r_K}{r_{eff}(S)} = \frac{A}{r_{eff}(S)}.$$

The second term on the right-hand side of equation (4) is

$$\ln a_w = \frac{B}{1 - g_{eff}^3(S)}$$

This implies,

$$a_w = \exp\left(\frac{B}{1 - g_{eff}^3(S)}\right) \quad (6)$$

The parameter B was described as the bulk hygroscopicity factor under sub saturation conditions

From equations (5) and (6), equation (1) for multiple components can be written as

$$S(g_{eff}, g_\sigma(r_{eff})) = a_w(g_{eff}) \exp[g_\sigma(r_{eff})] \quad (7)$$

The error to the first order on ambient RH due to the effect of Kelvin effect on equation (7) was determined by Lewis (2006) as:

$$\Delta S(g_{eff}, g_\sigma(r_{eff})) = S(g_{eff}, g_\sigma(r_{eff})) g_\sigma(r_{eff}) \quad (8)$$

The fractional change in the equilibrium pressure $S(g_{eff}, g_\sigma(r_{eff}))$ due to the error in Kelvin effect was determined as

$$\frac{\Delta S(g_{eff}, g_\sigma(r_{eff}))}{S(g_{eff}, g_\sigma(r_{eff}))} = \frac{S(g_{eff}, g_\sigma(r_{eff}))}{S(g_{eff}, g_\sigma(0))} g_\sigma(r_{eff}) \quad (9)$$

The lowest correction on the effective hygroscopic growth due to the Kelvin effect was also obtained by (Lewis, 2006) as:

$$\Delta g_{eff}(S, g_{\sigma}(r_{eff})) = -\frac{r_K}{r_{eff}(0,0)} \frac{S}{g_{eff}(S,0)} \frac{dg_{eff}(S,0)}{dS} \quad (10)$$

Similarly, the lowest-order correction to the effective radii due to the Kelvin effect was determined by (Lewis, 2006) as:

$$\Delta r_{eff}(S, g_{\sigma}) = -r_K \frac{S}{g_{eff}(S,0)} \frac{dg_{eff}(S,0)}{dS} \quad (11)$$

Important observation that can be made from equations (10) and (11) are that, as $r_{eff}(S, 0)$ and $g_{eff}(S, 0)$ are directly dependent on the ambient RH, the corrections will be negative, implying that the effective radii and the effective hygroscopic growth and their ratios calculated from the bulk solution properties will be overestimated.

3. MATERIALS AND METHODS

This part of the paper deal with methods and procedures used in conducting the study. It clearly showed the procedures used to carried out the analysis. The methodology adopted in the study combines aerosols microphysical data from OPAC 4.0, Koehler theory and multiple regression analysis to quantify the influence of the Kelvin effect and hygroscopicity on the Mineral Nucleation Non-Spherical (MINN) Mode of desert aerosols under different relative humidity (RH) conditions. The procedure is summarized below.

3.1 Data Collection

Microphysical properties (volume mix ratios, dry mode radii and wet mode radii) of desert aerosols were extracted from the OPAC 4.0 (Optical Properties of Aerosols and Clouds). Data were extracted over the spectral range of (0.4 to 0.8 μm) and at eight relative humidities of (0%, 50%, 70%, 80%, 90%, 95%, 98%, and 99%).

3.2 Data Processing

Using koehler theory the extracted inputs were used to compute: (i) Effective hygroscopic growth factor, $geff$ using equation 7, (ii) Effective radius, $reff$ using equation 8, (iii) water activity, aw using equation11, (iv) equilibrium saturation pressure, S using equation (12).

3.2.1 Regression Based on Kohler Equations: The linearized koehler equation (equation4),

$\ln S = \frac{A}{r_{eff}(S)} + \frac{B}{1-g_{eff}^3(S)}$, was used to obtain coefficients A (Kelvin Curvature term) and B (Solution term) for each model and R^2 using SPSS 14.0.

3.2.2 Parameterization of Hygroscopic Growth: Hygroscopic growth was further described using three empirical models:

i) γ -model (Eqn 17) ii) k-model and iii) the three-parameter growth model (Eqn. 24) to estimate the parameters: first order Kelvin correction, lowest correction on the effective

hygroscopic growth due to the Kelvin effect and the fractional change in the equilibrium pressure for each model across RH range.

3.3 Data Analysis and interpretation

Results were compared across models to assess the combined influence of RH and mineral composition on hygroscopic growth and kelvin curvature. Using spss 14.0 and the results of the output plot using origin 60.

3.4 Procedure

3.4.1 OPAC 4.0 Software Package

The OPAC desert aerosol type comprises water soluble particles and mineral particles in nucleation, accumulation and coarse modes. In the present simulations, the MINN component was varied while the other components were held constant. The five compositions used are summarized in Table 1.

Table 1: The models used in the simulations of the desert aerosols at 0% RH (MINN)

	Model1	Model2	Model3	Model4	Model5
Comp	No. Den(cm^{-3})				
WASO	2000	2000	2000	2000	2000
MIAN	30.50	30.50	30.50	30.50	30.50
MICN	0.142	0.142	0.142	0.142	0.142
MINN	269.5	350.35	431.2	512.05	592.9

3.4.2 KELVIN EFFECT

To parameterize the hygroscopic growth factor distribution of the aerosol, three equations models that were used by various authors in their researches to determine the relationship between the Kelvin effects and RH were adopted.

3.4.3 First model

It was employed to evaluate the Kelvin effect, the empirical γ -model, which has been widely used in the literature (Gysel *et al.*, 2009, Putaud, 2012, Tijjani, 2013a, Tijjani & Uba, 2013a, Tijjani & Uba, 2013b, Tijjani, 2013b, Tijjani *et al.*, 2013) to characterize the hygroscopic growth of atmospheric aerosol particles and is expressed as:

$$g_{eff}(S, 0) = (1 - S)^{-n} \quad (12)$$

where the values of n depend on the type of solutes and on the range of relative humidity.

To evaluate equations (10) and (11) for the model given by equation (12), the following relations which were derived as:

$$\Delta g_{eff}(S, g_{\sigma}(r_{eff})) = -\frac{r_K}{r_{eff}(0,0)} n \frac{S}{1-S} \quad (13)$$

and

$$\Delta r_{eff}(S, g_\sigma(r_{eff})) = -S r_K \frac{n}{1-S} \quad (14)$$

We determined the fractional changes in the effective hygroscopic growth and effective radii due to the Kelvin

effect from equations (18) and (19) as:

$$\frac{\Delta g_{eff}(S, g_\sigma(r_{eff}))}{g_{eff}(S, 0)} = -\frac{r_K}{r_{eff}(0, 0)} \frac{Sn(S)}{(1-S)^{1-n}} \quad (15)$$

and

$$\frac{\Delta r_{eff}(S, g_\sigma)}{r_{eff}(S, 0)} = -\frac{r_K}{r_{eff}(S, 0)} \frac{Sn(S)}{1-S} \quad (16)$$

3.4.4 Second model

This described the relation between $g_{eff}(S, 0)$ and S that has been parameterized in a good approximation by a one-parameter equation, proposed e.g. by (Petters & Kreidenweis, 2007), the k -model, (Gysel *et al.*, 2002) and used extensively by some researches (Tijjani, 2013a, Tijjani & Uba, 2013a, Tijjani & Uba, 2013b, Tijjani, 2013b, Tijjani *et al.*, 2013) as:

$$g_{eff}(S, 0) = \left(1 + \kappa \frac{S}{1-S}\right)^{\frac{1}{3}} \quad (17)$$

“The hygroscopic parameter κ is a quantity used commonly to describe the hygroscopicity of atmospheric aerosols. The coefficient κ is a simple measure of the particle's hygroscopicity and captures all solute properties (Raoult effect),

Evaluation of equations (10) and (11) for the model given by equation (17) we determined:

$$\Delta g_{eff}(S, g_\sigma(r_{eff})) = -\frac{Sk}{3(1-S)^2} \frac{r_K}{r_{eff}(0, 0)} \left(1 + k \frac{S}{1-S}\right)^{-1} \quad (18)$$

and

$$\Delta r_{eff}(S, g_\sigma) = -\frac{Sk r_K}{3(1-S)^2} \left(1 + k \frac{S}{1-S}\right)^{-1} \quad (19)$$

From equation (17) the fractional change in the effective hygroscopic growth factor and effective radii due to the Kelvin effect were determined using equations (18) and (19) as

$$\frac{\Delta g_{eff}(S, g_\sigma(r_{eff}))}{g_{eff}(S, 0)} = -\frac{Sk(S)}{3(1-S)^2} \frac{r_K}{r_{eff}(0, 0)} \left(1 + \frac{Sk(S)}{1-S}\right)^{-4/3} \quad (20)$$

$$\frac{\Delta r_{eff}(S, g_\sigma)}{r_{eff}(S, 0)} = -\frac{Sk(S) r_K}{3r_{eff}(S, 0)(1-S)^2} \left(1 + \frac{Sk(S)}{1-S}\right)^{-1} \quad (21)$$

3.4.5 Third model

It was used by (Stanier *et al.*, 2004) for the calculation of efflorescence branch humidigrams is called volume growth factor GF_{VOL} . Aerosol water content is a function of RH, chemical composition, and state of hydration (deliquescence branch or efflorescence branch).

For continuous water uptake systems measured by a HTDMA, according to (Kreidenweis *et al.*, 2005, Dick *et al.*, 2000) HTDMA GF values can be parameterized in terms of water activity using a three-parameter curve fit defined as:

$$g_{eff}(S, 0) = 1 + (a_1 + a_2 S + a_3 S^2) \frac{S}{1-S} \quad (22)$$

where S is the relative humidity, and a_1 , a_2 , and a_3 are adjustable parameters.

To evaluate equations (10) and (11) for the model given by equation (22), we used the following relations which were determined by us and (Lewis, 2006) as:

$$\Delta g_{eff}(S, g_\sigma(r_{eff})) = -\frac{Sr_K}{3r_{eff}(0, 0)(1-S)^2} \frac{(a_1 + a_2(2S - S^2) + a_3(3S^2 - 2S^3))}{g_{eff}^3} \quad (23)$$

and

$$\Delta r_{eff}(S, g_\sigma(r_{eff})) = -\frac{Sr_K}{3(1-S)^2} \frac{(a_1 + a_2(2S - S^2) + a_3(3S^2 - 2S^3))}{g_{eff}^3} \quad (24)$$

We determined the fractional changes in the effective hygroscopic growth and effective radii due to the Kelvin effect from equations (23) and (24) as:

$$\frac{\Delta g_{eff}(S, g_\sigma(r_{eff}))}{g_{eff}(S, 0)} = -\frac{r_K}{a r_{eff}(0)} \frac{nS}{(1-S)^{1-n}} \quad (25)$$

and

$$\frac{\Delta r_{eff}(S, g_\sigma)}{r_{eff}(S, 0)} = -\frac{r_K}{r_{eff}(S)} \frac{nS}{1-S} \quad (26)$$

4. RESULTS AND DISCUSSIONS

This section presents the application of Koehler theory on the three models used and discussed the resulting effects of relative humidity and MINN concentration.

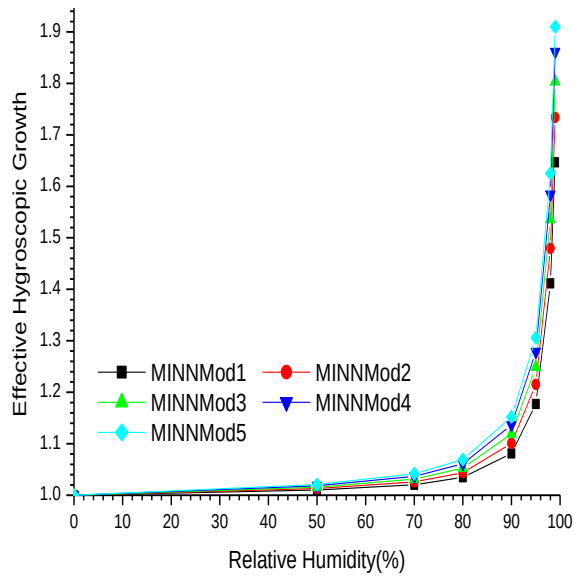


Figure 1: A plot of Effective Hygroscopic Growth of MINN Models against Relative Humidity using equation (2)

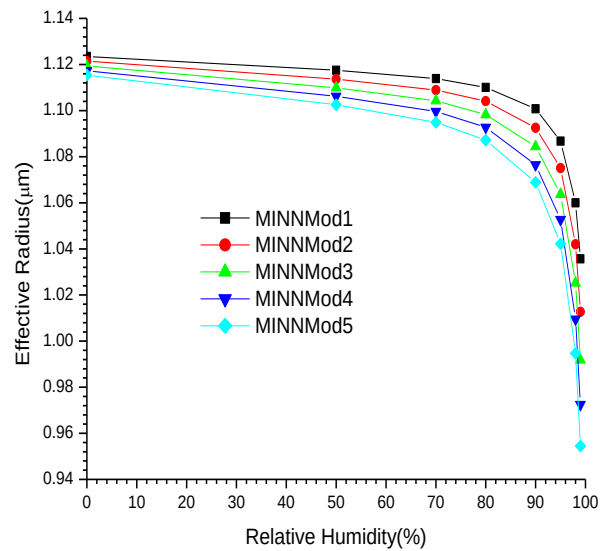


Figure 2: A plot of Effective Radius of MINN Models against Relative Humidity using equation (3)

Figure 1 illustrates the plot of the effective hygroscopic growth factor of the five MINN desert aerosol models with increasing relative humidity. The curves show that the effective hygroscopic growth of the models increases with increase in RH. All the models show a noticeable increase in their effective hygroscopic growth at 99% RH. Among the five models, Model 5 exhibits the highest hygroscopic growth, followed by Models 4, 3, 2 and 1. This suggests that the higher-numbered MINN models possess greater effective hygroscopicity, enabling them to take up more water vapour under humid conditions.

Figure 2 presents the plot variation of the equilibrium effective radius of the MINN models with relative humidity. The effective radius curves follow the same pattern and decreases more rapidly above 90% RH. MINN Model 1 maintains the largest effective radius throughout the humidity range, whereas Model 5 exhibits the smallest effective radius. The progressive decrease from Model 1 to Model 5 reflects the increasing influence of the Kelvin curvature effect.

Kelvin effect (MINN)

Table 2. Results of the regression analysis of MINN for equation (4)

Models	R ²	Sig	A	Sig	B	Sig
1	0.999055	2.75E-08	0.01819	0.028191	0.020852	3.88E-08
2	0.999124	2.27E-08	0.01696	0.030123	0.026850	3.14E-08
3	0.999167	2.00E-08	0.01589	0.033099	0.032755	2.71E-08
4	0.999224	1.68E-08	0.01481	0.035755	0.038553	2.22E-08
5	0.999293	1.33E-08	0.01373	0.038062	0.044261	1.73E-08

From Table 2, it can be seen from the values of R^2 and their corresponding significance values that the regression equation fitted the data very well. By observing the values of R^2 from model 1 to model 5 it can be seen that R^2 increases slightly from 0.99905 to 0.999293 while the significance values decrease from 2.7×10^{-8} to 1.33×10^{-8} . This indicates that equation (4) accurately represents Kelvin

effect for MINN desert aerosols. By observing the values of A, which represents the Kelvin radii, it is observed the values increases from -0.01819 to -0.01373. This suggests that the magnitude of the Kelvin curvature decrease as the MINN aerosol Model changes from Model 1- Model 5. The significance values associated with A increase slightly from 0.02819 to 0.03806, while the values of B (bulk

hygroscopicity factor) increase steadily from 0.02085 to 0.04426 across the MINN models. This indicates that the mineral nucleation aerosols exhibit an increasing tendency to absorb moisture. Such behavior is consistent with Koehler

theory. The significance values decrease from 3.88×10^{-8} to 1.73×10^{-8} , confirming that the hygroscopicity parameter remains highly significant throughout all MINN models.

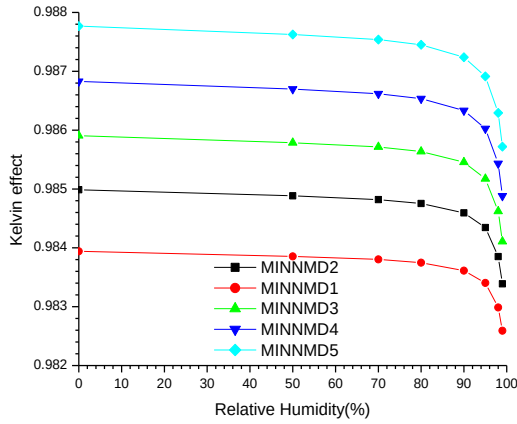


Figure 3: A plot of Kelvin Effect of MINN Models against Relative Humidity using equation (5)

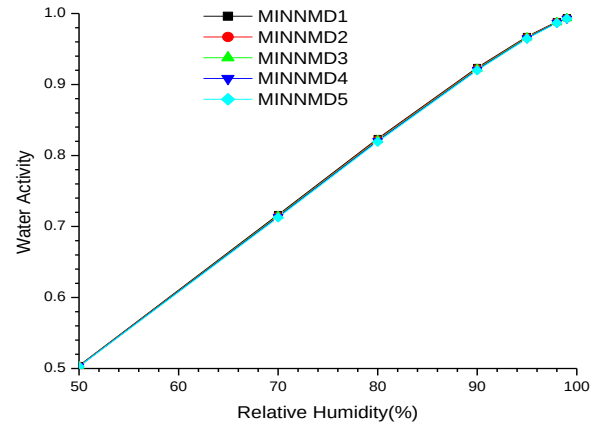


Figure 4: A plot of Water Activity of MINN Models against Relative Humidity using equation (6)

Figure 3 shows the plots of the Kelvin effect with relative humidity for the five MINN models. The Kelvin effect of all the models decreases slightly as relative humidity increases. Among the models, MINN Model 5 has the highest Kelvin effect, while Model 1 has the lowest values throughout the humidity range. This indicates that the influence of surface curvature becomes stronger as the aerosol hygroscopicity increases.

Figure 4 presents the plot variation of water activity with relative humidity for the MINN aerosol models. It can be observed that water activity increases almost linearly with increasing relative humidity, rising to nearly unity at 99% RH. The five MINN models show similar pattern, with water activity increasing with RH and approaching unity at 99% RH.

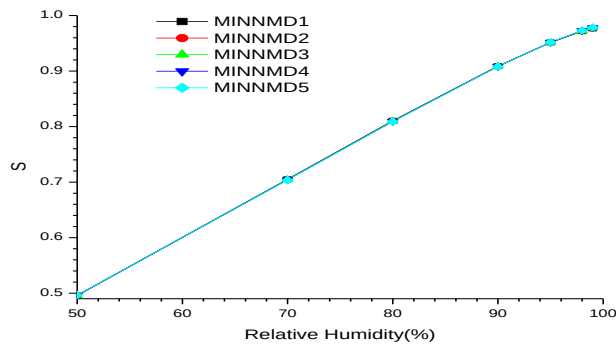


Figure 5: A Plot of $S(RH)$ of MINN Models against Relative Humidity using equation (7)

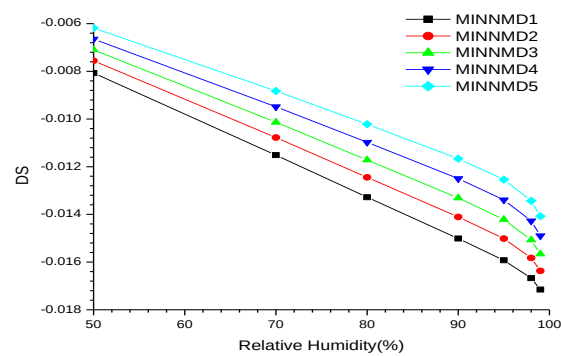


Figure 6: A plot of DS of MINN Models against Relative Humidity using equation (8)

Figure 5 illustrates the plot variation of the equilibrium vapour pressure with relative humidity. The curves show that all the aerosol models are highly sensitive to RH and the values approaching unity at 99% RH. All five MINN models produce almost identical curves, indicating that the equilibrium vapour pressure is practically independent of the aerosol model.

Figure 6 shows the plot variation of the first-order error in the equilibrium vapour pressure (DS) due to the Kelvin effect for the MINN desert aerosol models. The curve shows the same behavioural pattern. As the relative humidity increases, the aerosol models become more negative, decreasing from about -0.008 at 50% RH to approximately -0.017 at 99% RH.

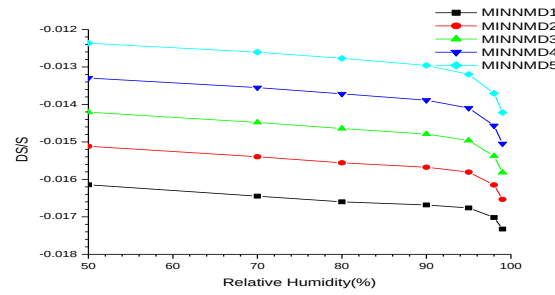


Figure 7: A plot of DS/S of MINN Models against Relative Humidity using equation (9)

Figure 7 presents the plot variation of the fractional change in the relative humidity (DS/S) due to the Kelvin effect for the MINN desert aerosol models. The figure shows that DS/S remains negative over the entire humidity range,

indicating that the Kelvin effect reduces the saturation ratio value. The curves decrease only gradually between 50% and about 95% RH but become noticeably steeper as RH approaches 99%.

First model

Table 3. Results of the regression analysis of the first model (equation 12)

Models	R2	Sig	n	Sig
1	0.870766	7.10E-04	0.079536	7.10E-04
2	0.870120	7.21E-04	0.079131	7.21E-04
3	0.869487	7.32E-04	0.078710	7.32E-04
4	0.868891	7.42E-04	0.078304	7.42E-04
5	0.868296	7.52E-04	0.077905	7.52E-04

From table 3, the values of R^2 indicate that equation (12) provides a good fit to the MINN aerosol data. It can be seen that the values of R^2 decreases slightly from 0.870766 to 0.868296, indicating a small reduction of the regression as MINN aerosol model changes. The significance values increase slightly from 7.10×10^{-4} to 7.52×10^{-4} , showing that

although the regression remains significant, the fit becomes weaker across the models. The regression coefficient n decreases gradually from 0.079536 to 0.077905. This shows that the parameter governing equation (12) becomes smaller as MINN vary from Model 1 to Model 5.

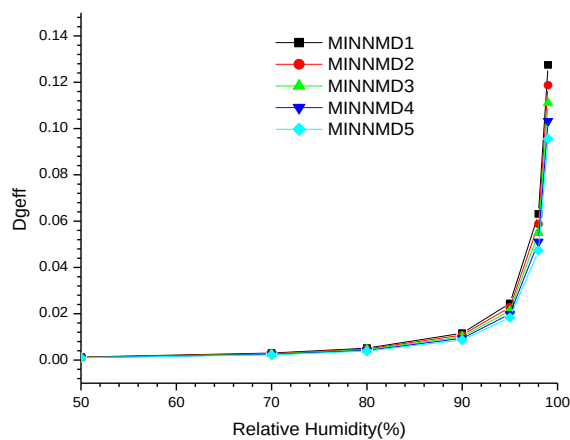


Figure 8: Plots of the lowest corrections on g_{eff} against RH, first model using equation (13)

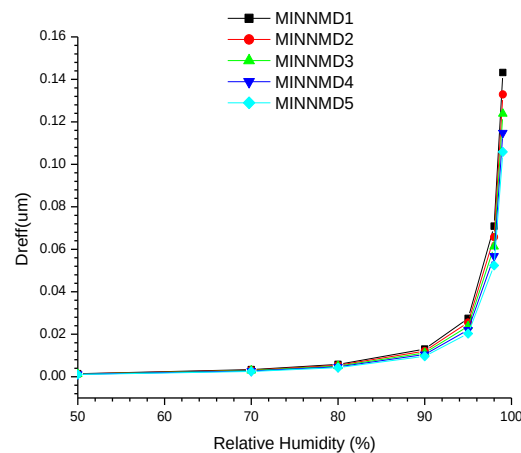


Figure 9: Plots of the lowest corrections on r_{eff} against RH, first model using equation, using equation (14)

From Figure 8, it can be observed that, the lowest corrections on g_{eff} are positive, implying that the effective hygroscopic growth calculated from the bulk solution properties were underestimated. The increase in the magnitude of the fractional change implies increase in the underestimation of the changes with the increase in both RH and MINN concentrations. The increase in the underestimation is more sensitive at higher RH 90 to 99%.

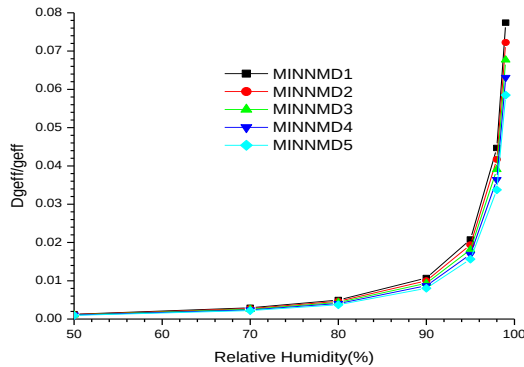


Figure 10: A plot of fractional change in the effective hygroscopic growth ($D_{\text{geff}}/g_{\text{eff}}$) of MINN Desert aerosols Models against Relative Humidity using equation (15)

Figure 10 shows the plot of fractional change in the effective hygroscopic growth ($D_{\text{reff}}/r_{\text{eff}}$) caused by the Kelvin effect. The models have similar curves pattern, indicating that the Kelvin effect contributes very little to the relative particle size correction under dry conditions. Beyond 90% RH, the fractional correction increases rapidly, with the largest changes occurring between 97% and 99% RH. The ordering of the models remains model1 > model2 > model3 > model4 > model5, indicating that Model 1 has the strongest correction, whereas Model 5 has the weakest.

MINN Second Model

Table 4. Results of the regression analysis of second model (equation 17)

Models	R^2	Sig	k	Sig
1	0.999128	2.08E-10	0.035264	2.08E-10
2	0.999132	2.04E-10	0.034983	2.04E-10
3	0.999131	2.05E-10	0.034689	2.05E-10
4	0.999137	2.01E-10	0.034408	2.01E-10
5	0.999132	2.04E-10	0.034130	2.04E-10

From Figure 9, it can be observed that, the lowest corrections on r_{eff} are positive, implying that the effective radii calculated from the bulk solution properties were underestimated. The increase in the magnitude of the fractional change implies increase in the underestimation of the changes with the increase in both RH and MINN concentrations. The increase with RH is more sensitive above RH higher than 90%. MINN model1 exhibits the largest correction in effective radius, whereas MINN model 5 gives the smallest correction.

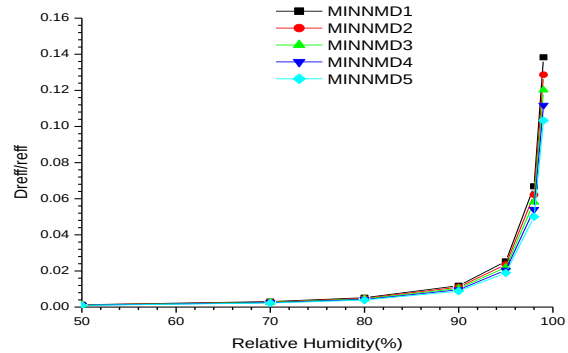


Figure 11: A plot of fractional change in the effective radii ($D_{\text{reff}}/r_{\text{eff}}$) of MINN Desert aerosols Models against Relative Humidity using equation (16)

Figure 11 shows the plot of the fractional change in the effective radii ($D_{\text{reff}}/r_{\text{eff}}$) of equation (22). From the plots, it can be observed that as the relative humidity increases from 80 to 90%, a gradual increase in the fractional change in the effective radii is observed for all the models. Although the correction remains small. Beyond 90% RH, the fractional change increases rapidly, with sharp rise between 95% and 99% RH.

From Table 4, it can be seen that R^2 ranges from 0.999128 to 0.999137, demonstrating that equation (17) accurately represents the MINN aerosol data. The corresponding significance values are on the order of 10^{-10} , showing an excellent statistical significance of regression. The regression coefficient k decreases gradually from 0.035264

in model1 to 0.034130 in model 5. This indicates that the parameter controlling equation (17) becomes slightly smaller across the MINN aerosol models. The values associated with k remain nearly constant throughout all models, indicating that the parameter is reliable and statistically stable.

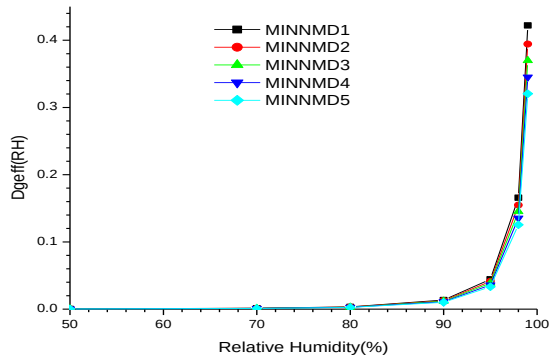


Figure 12: A plot of the correction on the effective hygroscopic growth ($D_{\text{geff}}(\text{RH})$) of MINN Desert aerosols Models against Relative Humidity using equation (18)

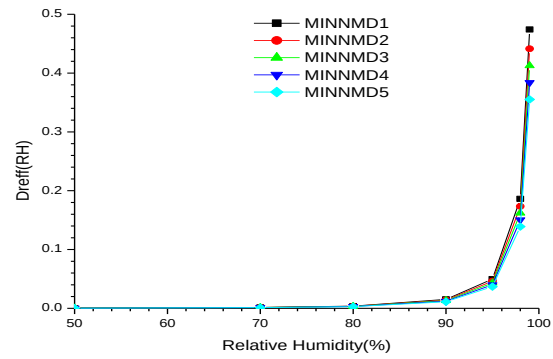


Figure 13: A plot of $D_{\text{reff}}(\text{RH})$ of MINN Desert aerosols Models against Relative Humidity using equation (19)

Figure 12 shows the plots of the correction to the effective hygroscopic growth ($D_{\text{geff}}/\text{geff}$) resulting from the Kelvin effect for the MINN desert aerosol models. The figure shows that the correction remains very small below approximately 90% RH, indicating that Kelvin curvature has little influence on hygroscopic growth under moderate humidity conditions. However, the curves rise rapidly above 95% RH.

Figure 13 shows the plot for the correction of effective radius due to the Kelvin effect, $\Delta_{\text{reff}}(\text{RH})$, for the MINN desert aerosol models as relative humidity (RH) increases. The figure shows that $\Delta_{\text{reff}}(\text{RH})$ remains very small between 50% and about 90% RH for all MINN models, indicating that the Kelvin curvature effect has very little influence on the effective radius under relatively dry atmospheric conditions. Above approximately 90% RH, $\Delta_{\text{reff}}(\text{RH})$ increases rapidly for all models.

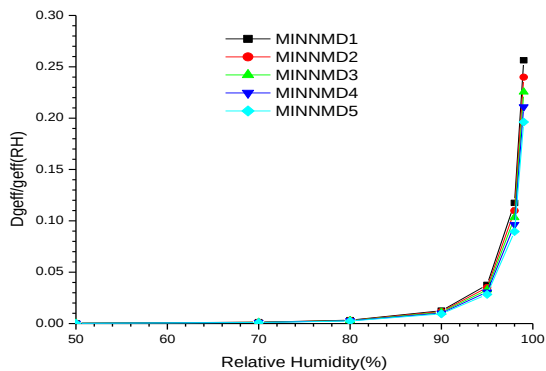


Figure 14: A plot of fractional change in the effective hygroscopic growth ($D_{\text{geff}}/\text{geff}(\text{RH})$) of MINN Desert aerosols Models against Relative Humidity using equation (20)

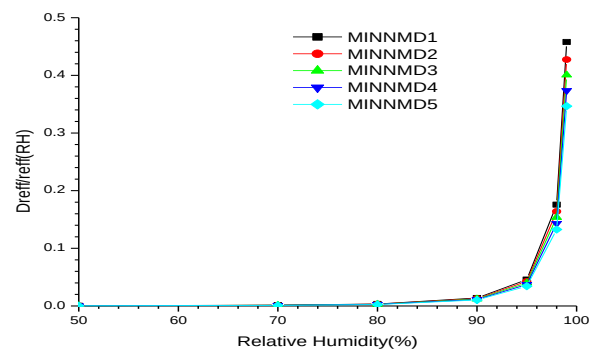


Figure 15: A plot of fractional correction in effective radii, ($D_{\text{reff}}/\text{reff}(\text{RH})$) of MINN Desert aerosols Models against Relative Humidity using equation (21)

Figure 14 illustrates the plot of the fractional change in the effective hygroscopic growth factor due to the Kelvin effect, $\Delta g_{eff}/g_{eff}(RH)$, for the MINN desert aerosol models. From the plots it can be observed that MINN model1 exhibits the highest underestimation, while MINN model5 has the lowest underestimation. This indicates that the effect of particle curvature becomes increasingly important during rapid water uptake at high humidity.

Third Model

Table 5. The regression analysis of third model (equation 22)

Models	R ²	Sig	a ¹	Sig	a ²	Sig	a ³	Sig
1	.9992550	.0000	.0202	.2697	.0000	.0000	0.0155	0.3972
2	.9992660	.0000	.0197	.2745	.0000	.0000	0.0158	0.3830
3	.9992710	.0000	.0192	.2801	.0000	.0000	0.0159	0.3726
4	.9992820	.0000	.0187	.2839	.0000	.0000	0.0161	0.3602
5	.9992820	.0000	.0183	.2896	.0000	.0000	0.0163	0.3534

From Table 5, it can be observed that the values of R² increase slightly from 0.999255 in model 1 to 0.999282 in model 5, indicating an excellent agreement between equation (22) and MINN aerosol data. The very small significance values further confirm the reliability of the regression model. The coefficient a¹ decreases gradually from 0.0202 to 0.0183, suggesting that the first-order correction of equation (22) becomes smaller as the MINN Models changes from Model 1 to Model 5. This behavior

Figure 15 shows the plot of the fractional correction in effective radii, $\Delta r_{eff}/r_{eff}(RH)$, for the MINN desert aerosol models. From the plots, it can be observed that MINN model1 has the highest underestimation followed by MINN model2, then MINN model3 followed by MINN model4 and MINN model5 has the lowest underestimation. All the overestimation and underestimation increased with RH in almost exponential or power form.

indicates that the first-order Kelvin correction contributes less to the effective radii across the MINN models. The significance values associated with a¹ increase from 0.2697 to 0.2896, indicating a small reduction in its statistical influence. The coefficient a³ increases from 0.0155 to 0.0163, suggesting that the higher-order contribution becomes more across the MINN models. At the same time, its significance values decrease from 0.3972 to 0.3534.

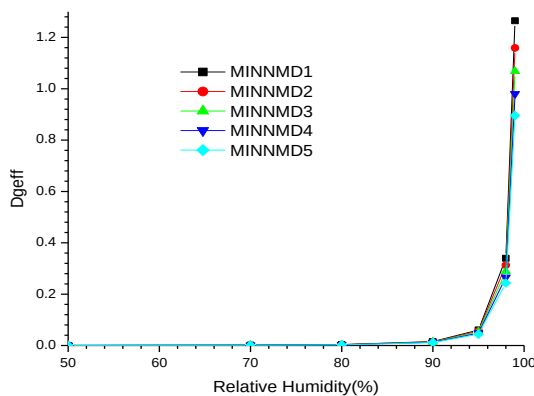


Figure 16: A plot of Dgeff of MINN Desert aerosols Models against Relative Humidity using equation (23)

Figure 16 presents the plot of the correction on the effective hygroscopic growth factor, Δg_{eff} , as a function of relative humidity. The correction remains negligible below 90% RH but increases rapidly as RH approaches 99% RH. The

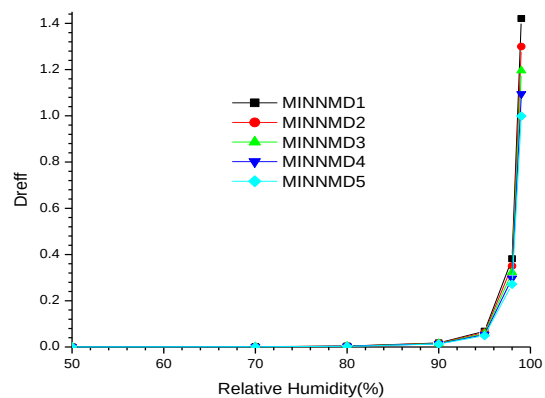


Figure 17: A plot of Dreff of MINN Desert aerosols Models against Relative Humidity using equation (24)

increase reflects enhanced water uptake by the aerosol particles under humid conditions, making the Kelvin effect increasingly important in determining hygroscopic growth.

Figure 17 illustrates the plot of the correction on effective radius, Δr_{eff} , of the MINN desert aerosol models. The correction remains very small throughout the lower humidity range but increases sharply after approximately 90% RH. The rapid increase near saturation indicates that water condensation becomes sufficiently strong for the Kelvin effect to produce noticeable changes in particle radius. Such behaviour demonstrates that the influence of

particle curvature becomes increasingly important during the final stages of hygroscopic growth.

MINN Model 1 shows the greatest correction, whereas MINN Model 5 exhibits the smallest correction. This suggests that increasing mineral concentration suppresses the magnitude of Kelvin effect radius corrections because mineral particles exhibit weaker hygroscopic growth than more soluble aerosol particles.

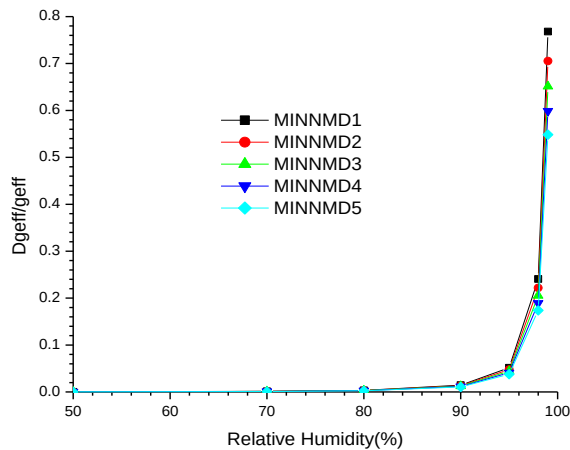


Figure 18: A plot of the fractional correction on the effective hygroscopic growth (D_{geff}/g_{eff}) of MINN Desert aerosols Models against Relative Humidity using equation (25)

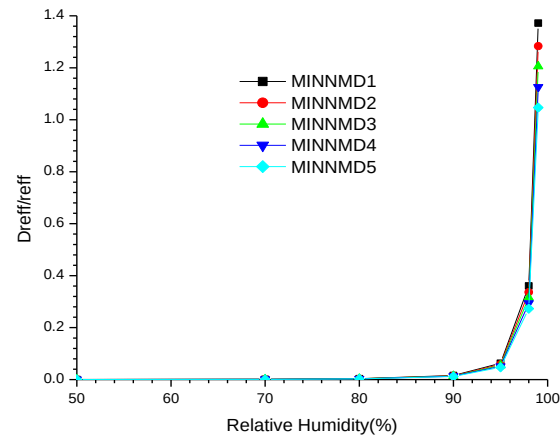


Figure 19: A plot of fractional correction on the effective radius (D_{reff}/r_{eff}) of MINN Desert aerosols Models against Relative Humidity using equation (26)

Figure 18 presents the plot of the fractional correction on the effective hygroscopic growth factor due to the Kelvin effect. The plot shows increase in the underestimation for the five aerosol models. The curves indicate that hygroscopic growth becomes increasingly sensitive to Kelvin curvature. The model ranking remains model1 > model2 > model3 > model4 > model5 throughout the humid region.

Figure 19 shows the plot of the fractional correction on the effective radius due to the Kelvin effect. The fractional change of all the models increase with increase in relative humidity. All aerosol models show appreciable increases in the fractional correction above 90% RH. Among the models, MINN Model 1 consistently produces the highest fractional correction, while MINN Model 5 produces the lowest.

5. CONCLUSION

This study investigated the hygroscopic behaviour and Kelvin effect of Mineral Nucleation Mode (MINN) of desert aerosols using OPAC 4.0, Koehler theory and three hygroscopic growth parameterizations (γ -model, Koehler model, and three-parameter model). The results showed that hygroscopic growth increased as relative humidity increased.

The effective radius decreased as concentration of mineral aerosols increased from Model1 to model5. Water activity increased with relative humidity and approached unity and equilibrium vapour pressure increase steadily with relative humidity. The Kelvin effect decreased slightly with increasing relative humidity. The first order and fractional Kelvin corrections remained negligible under moderate humidity conditions but increased rapidly between 95% and 99% relative humidity. Although all three parameterizations produced consistent trends, Model 1 showed the largest Kelvin Effect corrections, whereas Model 5 showed the smallest. The bulk regression gave $R^2 = 0.99905$ to 0.99929 , $A = 0.01819$ to 0.01373 and $B = 0.02085$ to 0.04426 . The κ -model produced $R^2 = 0.99912$ – 0.99913 , while the three-parameter model produced $R^2 = 0.99925$ to 0.99928 . The three growth models gave similar trends, showing good agreement with the simulated results.

These findings are consistent with previous studies based on Koehler theory and aerosol hygroscopic growth, including Hess et al. (1998), Petters and Kreidenweis (2007), Topping et al. (2005), Wex et al. (2008), Tijjani et al. (2015), and Rastak et al. (2017), thereby supporting the validity of the present approach. The study demonstrates that both relative humidity and mineral aerosol composition are key factors

controlling the hygroscopic and equilibrium properties of desert aerosols.

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