

THEORETICAL FORMULATION OF DELTA-T

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Abstract: Delta-T is the difference between Terrestrial Time and Universal Time. Terrestrial Time or Dynamical Time is a function of Earth's orbital motion. Universal time is a function of Earth's spin motion. Earth's spin motion experiences a secular lengthening due to tidal drag exerted by our Moon, it experiences periodic variation due to Saros Period (an interval of 18 years and 111/3 days after which the Earth, Moon and Sun return to nearly the same relative positions and cycle of lunar and solar eclipses begin to repeat itself), Earth's spin motion experiences periodic variation due to sidereal period of Moon, due to Spring and Neap tide. Periodic variations in Earth's spin take place due to seasonal changes also. Earth's spin motion experience irregular variation due to geophysical and geological factors such as plate tectonics, setting of an Ice-Age, thawing of ice-age, ocean currents and global wind patterns interacting with mountain ranges and electromagnetic coupling between core and mantle. UT1 or Universal Time is counted from 0 hour- 0 minute- 0 second midnight on 1/01/1900 is measured in terms of spin period. Over long term the periodic and irregular variations will average out but secular lengthening of the length of the day will continue as a result UT1 will fall behind the dynamical time. When UT1 falls behind TT by more than 0.7 second then the clock is stopped for 1 second and this is called addition of a leap second. The addition of 1 leap second or stopping the clock by 1 second advances UT1 by 0.3 second. ahead of TT. This creates the need for Coordinated Universal Time (UTC). UTC keeps the record of the number of leap seconds added. In this paper Delta-T is measured by integrating the deficit to the day as defined in 1900 CE (Current Era) from ancient days to 1900CE. This is compared with Delta-T as observed by Stephenson and Stephenson & Houldon (1896). The analytical values calculated by the Author match with the observed values except for the intermediate periods. This is explained by taking into account the post-glacial rebound taking place between 2000BCE and 1600CE. This match stands as a vindication of the new perspective on the birth and evolution of Solar System.

Keywords: Universal Time; Terrestrial Time; Atomic Time; Leap Seconds; Post Glacial Rebound.

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1. Introduction

Sharma [1995] at the Silver Jubilee Anniversary Celebration of Man's landing on Moon had given the time integral equation for the spiral trajectory of Moon starting at Roche's Limit of 15,700 km as given by Ida, Canup and Steward [Ida et al 1997] to the present Lunar Orbital Radius of 3,84,400 km with the primary bench mark of 4.53 billion years as the present age of Moon. Sharma & Ishwar [2002] gave Time Integral Equation as follows:

$$(t_p - t_0) \text{ Solar Years} = \int [2.309982E22 (r_L^{(M-1/2)} - 5*8.996706829E-21* r_L^{(M+3/2)} + 7*8.996706829E-21* r_L^{(M+5/2)}) / (5.568961629E8) / (6.002958088E25 - 1.036806853E8* r_L^{(M+1)})] \text{ where } M = 2.1; \quad (2.1)$$

The above definite integral integrated from 1.8E7 meter, which is the orbital radius at which the Moon was placed at time of

accretion, also known as Roche's Limit [Ida et al 1997], to 3.844E8 meter.

From Equation (2.1) theoretical values of lunar orbital radii are calculated for different geologic epoch. From this set of data, an empirical relationship for lunar orbital radial expansion is obtained as follows [Sharma & Ishwar 2002]

$$r_L = 5.569E8 - (5.569E8 - 0.18E8) \text{ Exp } [-t/2E9] - 1.6167E8 \text{ Exp } [-t/14.1E9] + 1.6E8 \text{ Exp } [-t/1E9] \quad (2.2)$$

Sharma & Ishwar [ibid] gave the expression for length of day as a function of lunar orbital radius r_L :

$$T_E \text{ (length of day)} = [2\pi C/3600] / [M - (B/X^2)(D + EX^2)Y'] \quad (2.3)$$

Where $M = 3.440488884E34 \text{ Kg} - m^2/\text{sec} = \text{total scalar angular}$

Momentum of Earth – Moon System;

$C = \text{principal moment of inertia of Earth around polar}$

$$\text{Axis} = 8.02E37 \text{ Kg} - m^2;$$

$$B = 2.008433303E7 \text{ m}^{3/2}$$

$X = (r_L) = \text{Lunar Orbital Radius of the given geological epoch};$

$$D = 8.878598241E34 \text{ Kg} - m^2;$$

$$E = 7.258980539E22 \text{ Kg};$$

$$Y' = (X^{1/2} - AX^{5/2} + AX^{7/2}/Y);$$

$$A = 8.996706829 \text{ E} - 21 \text{ m}^{-2};$$

$Y = \text{final limit of the Lunar Orbital Radius when Moon will be in Geo-synchronous orbit} = (r_L)_f = 5.568961629 \text{ E}8 \text{ m}.$

Using Mathematica Command, length of day is calculated for the present epoch when Lunar Orbital Radius is 3.844E8 meter.

First Lunar Orbital Expression Eq.(2.2) is written in Mathematica Command and checked at the present epoch.

$$\begin{aligned} & 5.569 \times 10^8 - (5.569 \times 10^8 - 0.18 \times 10^8) \text{Exp}[-t \div (2 \times 10^9)] - \\ & 1.6167 \times 10^8 \times \text{Exp}[-t \div (14.1 \times 10^9)] + \\ & 1.6 \times 10^8 \times \text{Exp}[-t \div (1 \times 10^9)] \text{ /. } t \rightarrow 4.5 \times 10^9 \end{aligned}$$

(2.2)

At the present time the answer is:

$$3.8438132805073927^{*}8 \text{ m as the Lunar Orbital Radius should be.}$$

Next the length of day Eq. (2.3) is written except that x is replaced by the expression in Eq. (2.2):

$$\begin{aligned} & ((2\pi \times 8.02 \times 10^{37}) \div 3600) \div \\ & (3.440488884 \times 10^{34} - ((2.008433303 \times 10^7) \div x^2) \\ & (8.878598241 \times 10^{34} + (7.258980539 \times 10^{22}) \times x^2) \times \\ & (x^{0.5} - 8.996706829 \times 10^{-21} \times x^{2.5} + \\ & 8.996706829 \times 10^{-21} \times x^{3.5} \div (5.568961629 \times 10^8))) \text{ /. } \\ & x \rightarrow 5.569 \times 10^8 - (5.569 \times 10^8 - 0.18 \times 10^8) \text{Exp}[-t \div (2 \times 10^9)] - \\ & 1.6167 \times 10^8 \times \text{Exp}[-t \div (14.1 \times 10^9)] + 1.6 \times 10^8 \times \text{Exp}[-t \div (1 \times 10^9)] \end{aligned}$$

The result of evaluating the kernel is:

$$\begin{aligned} & 1.39975406009945203^{*}35 / (3.440488884000000052^{*}34 - \\ & (2.0084333029999994^{*}7 (8.87859824100000238^{*}34 + \\ & 7.25898053900000128^{*}22 (5.5689999999999995^{*}8 + \\ & 1.60000000000000008^{*}8 \text{E}^{-t/1000000000} - \\ & 5.38900000000000023^{*}8 \text{E}^{-t/2000000000} - \\ & 1.61670000000000024^{*}8 \\ & \text{E}^{-7.09219858156028415^{*}11 t})^{\wedge} \\ & 2) \\ & ((5.5689999999999995^{*}8 + \\ & 1.60000000000000008^{*}8 \text{E}^{-t/1000000000} - \\ & 5.38900000000000023^{*}8 \text{E}^{-t/2000000000} - \\ & 1.61670000000000024^{*}8 \text{E}^{-7.09219858156028415^{*}11 t})^{\wedge} \\ & 0.5 - \end{aligned}$$

$$\begin{aligned}
 & 8.99670682899999896 \times 10^{-21} (5.5689999999999995 \times 10^8 + \\
 & 1.60000000000000008 \times 10^8 E^{-t/1000000000} - \\
 & 5.38900000000000023 \times 10^8 E^{-t/2000000000} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-7.09219858156028415 \times 10^{-11} t})^{2.5} + \\
 & 1.61550885575333147 \times 10^{-29} (5.5689999999999995 \times 10^8 + \\
 & 1.60000000000000008 \times 10^8 E^{-t/1000000000} - \\
 & 5.38900000000000023 \times 10^8 E^{-t/2000000000} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-7.09219858156028415 \times 10^{-11} t})^{3.5}) / \\
 & (5.5689999999999995 \times 10^8 + \\
 & 1.60000000000000008 \times 10^8 E^{-t/1000000000} - \\
 & 5.38900000000000023 \times 10^8 E^{-t/2000000000} - \\
 & 1.61670000000000024 \times 10^8 E^{-7.09219858156028415 \times 10^{-11} t})^A \\
 & 2)
 \end{aligned}$$

(2.4)

Equation (2.4) is the theoretical expression for lod as a function of time since the Giant Impact. This when evaluated at $t=4.5$ billion years gives the length of day of 23.997 hours.

Equation (2.4) becomes the basis for evaluating Delta-T

2. WHAT IS DELTA-T

Delta-T or $\Delta T = TT - UT1$

Where TT is Terrestrial Time or Dynamical Time. Regular motions such as Earth's Orbital Motion is a function of Dynamical Time whereas Earth's Spin Motion experiences variation hence is not a function of Dynamical time.

Earth's Spin Motion experiences a secular deceleration due to Earth-Moon interaction as has been shown by Sharma [1995] and Sharma & Ishwar [2002];

It experiences periodic variation due to tides at a period of 18.08 years (Saros Period), 1 year, $\frac{1}{2}$ year, sidereal period of Moon = 27.3 days due to Spring and Neap tide and at a period of 13.66 days the phase difference between Spring and Neap tides [Leschiutta & Tavella 2001, Yan et al 2002, Stephenson 2003]. This has been reported by IERS (www.iers.org/ Astronomical times);

Seasonal changes cause periodic variation of 1 year and $\frac{1}{2}$ year period [ibid].

Earth's Spin Motion also experiences irregular variation due to geophysical and geographical factors such as plate tectonics, setting of ice-age, thawing of ice age, ocean currents and global wind patterns interacting with mountain ranges and electromagnetic coupling between core and mantle [ibid]. Whatever factor increases the Principal Moment of Inertia around the polar axis [C] in real term or apparent term slows down the spin in addition to the secular deceleration. The factor which decrease C in real or apparent term hasten the spin in opposition to the secular slowing down of the spin.

Abstract of the article written by Yan et al [2002] "The interannual changes in the Earth's rotation rate, and hence in the

length of day (LOD), are thought to be caused by the variation of the atmospheric angular momentum (AAM). However, there is still a considerable portion of the LOD variations that remain unexplained. Through analyzing the non-atmospheric LOD excitation contributed by the Western Pacific Warm Pool (WPWP) during the period of 1970–2000, the positive effects of the WPWP on the interannual LOD variation are found, although the scale of the warm pool is much smaller than that of the solid Earth. These effects are specifically intensified by the El Niño events, since more components of the LOD-AAM were accounted for by the warm pool excitation in the strong El Niño years. Changes in the Earth's rotation rate has attracted significant attention, not only because it is an important geodetic issue but also because it has significant value as a global measure of variations within the hydrosphere, atmosphere, cryosphere and solid Earth, and hence the global changes."

UT1 or Universal Time counted from 0 hour-0 minute-0 second on 1/01/1900 is measured in terms of Earth's spin period. Over long term the periodic and irregular variations may average out but secular lengthening of length of day will continue as a result UT1 will fall behind the Dynamical Time.

When UT1 falls behind TT by more than 0.7 second then the clock is stopped for 1 second. This is called addition of a Leap Second and usually takes place at an interval of 20 months either 0-0-0 of 30th June or 31st December. The addition of 1 Leap Second or stopping the Clock for 1 second advances UT1 by 0.3 second ahead of TT.

This creates the need for Coordinated Universal Time (UTC). UTC keeps the record of the number of Leap Seconds added.

The standard SECOND is defined as $1/86400$ part of the day which occurred on 1/01/1900 or

It is also defined as the time elapsed when 9,192,631,770 cycles have occurred in the radiation corresponding to transition in Cesium-133 between two hyper-fine levels in the ground state.

Time in terms of the standard second is being measured by Cesium Clock and is being counted since 0-0-0 of 1/01/1958. This is known as TAI or International Atomic Time.

TAI = UTC at 0-0-0 of 1/01/1958.

On 0-0-0 of 1/01/1977, TT = TAI + 32.184 seconds.

Also UTC = TAI + n* (1 Leap Second) where n* = integer number.

Therefore 0-0-0 of 1/01/1900 is the reference point for measuring $\Delta - T$.

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On 0-0-0 of 1/01/1977, TT = TAI + 32.184 seconds.

Also UTC = TAI + n* (1 Leap Second) where n = integer number.

Therefore 0-0-0 of 1/01/1900 is the reference point for measuring $\Delta - T$.

$\Delta - T = \text{Integrate } [(86400 - \text{actual length of day}) dt, \text{ancient time in days, (1900 AD) present time in days}] = \Delta - T \text{ of the given ancient time.}$

Here one point needs to be emphasized namely:

(86400-actual length of day in seconds) gives the deficit to 86400 seconds per day therefore $\Delta - T$ will be correctly evaluated only when the integration is carried out with respect to days and not with respect to years elapsed since the Giant Impact. Therefore Lunar Orbital Expansion expression as well as the deficit to 86400 seconds expression needs to be expressed in days elapsed since the Giant Impact in order to obtain the correct result for $\Delta - T$.

$$5.569 \times 10^8 - (5.569 \times 10^8 - 0.18 \times 10^8) \text{Exp}[-t \div (2 \times 10^9 \times 365.2468)] - \\ 1.6167 \times 10^8 \times \text{Exp}[-t \div (14.1 \times 10^9 \times 365.2468)] + \\ 1.6 \times 10^8 \times \text{Exp}[-t \div (1 \times 10^9 \times 365.2468)] / . t \rightarrow 4.5 \times 10^9 \times 365.2468$$

(2.5)

The above expression is the Lunar Orbital Radius Expansion expression in days elapsed since the Giant Impact. When calculated at $t = 4.5E9 \times 365.2468$ days since the Giant Impact it gives the correct result of $3.844E8$ m, the present lunar orbital radius..

Eq. (2.5) is substituted in lod expression Eq (2.3). Thus we obtain length of day expression as a function of time in days elapsed since the Giant Impact. It is given as follows:

$$\begin{aligned}
 & \frac{1.39975406009945203 \times 10^{35}}{(3.44048888400000052 \times 10^{34} -} \\
 & (2.00843330299999944 \times 10^7 (8.87859824100000238 \times 10^{34} + \\
 & 7.25898053900000128 \times 10^{22} \\
 & (5.5689999999999995 \times 10^8 + \\
 & 1.6000000000000008 \times 10^8 \\
 & E^{-2.73787477398843748 \times 10^{-12} t} - \\
 & 5.38900000000000023 \times 10^8 \\
 & E^{-1.36893738699421874 \times 10^{-12} t} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-1.94175515885704754 \times 10^{-13} t})^2) \\
 & ((5.5689999999999995 \times 10^8 + 1.6000000000000008 \times 10^8 \\
 & E^{-2.73787477398843748 \times 10^{-12} t} - \\
 & 5.38900000000000023 \times 10^8 \\
 & E^{-1.36893738699421874 \times 10^{-12} t} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-1.94175515885704754 \times 10^{-13} t})^{0.5} - \\
 & 8.99670682899999896 \times 10^{-21} \\
 & (5.5689999999999995 \times 10^8 + \\
 & 1.6000000000000008 \times 10^8 \\
 & E^{-2.73787477398843748 \times 10^{-12} t} - \\
 & 5.38900000000000023 \times 10^8 \\
 & E^{-1.36893738699421874 \times 10^{-12} t} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-1.94175515885704754 \times 10^{-13} t})^{2.5} + \\
 & 1.6155088557533147 \times 10^{-29} \\
 & (5.5689999999999995 \times 10^8 + \\
 & 1.6000000000000008 \times 10^8 \\
 & E^{-2.73787477398843748 \times 10^{-12} t} - \\
 & 5.38900000000000023 \times 10^8 \\
 & E^{-1.36893738699421874 \times 10^{-12} t} - \\
 & 1.61670000000000024 \times 10^8 \\
 & E^{-1.94175515885704754 \times 10^{-13} t})^{3.5})) / \\
 & (5.5689999999999995 \times 10^8 + \\
 & 1.6000000000000008 \times 10^8 E^{-2.73787477398843748 \times 10^{-12} t} - \\
 & 5.38900000000000023 \times 10^8 E^{-1.36893738699421874 \times 10^{-12} t} - \\
 & 1.61670000000000024 \times 10^8 E^{-1.94175515885704754 \times 10^{-13} t})^2) / \\
 & 2) / . \\
 & t \rightarrow 4.500534082 \times 10^9 \times 365.2468
 \end{aligned}$$

(2.6)

As already shown if 1900 of the current epoch is taken as $4.500534082E9 \times 365.24268$ days elapsed since the Giant Impact then Length of Day is evaluated from the above expression to be: **23.9999999925531914** days.

Rewriting the above expression as deficit to 86400 seconds in seconds per day we get the following expression :

Eq.(2.7) is the expression for deficit to 86400 seconds. In the present epoch on 1/01/1900 works deficit is **0.0000268085074139889911** second per day. This expression is the needed expression which needs to be integrated from the ancient times to 1900 of the current epoch which we have taken as $t = 4.500534082E9 \times 365.2468$ days since the Giant Impact. The form of integration is the following:

$$\begin{aligned}
 & \text{NIntegrate}\left[3600 \left(24 - 1.39975406009945203 \cdot 10^{35} / \right. \right. \\
 & \quad \left. \left(3.44048888400000052 \cdot 10^{34} - (2.00843330299999944 \cdot 10^{47} \right. \right. \\
 & \quad \left. \left(8.87859824100000238 \cdot 10^{34} + \right. \right. \\
 & \quad \left. \left. 7.25898053900000128 \cdot 10^{22} \right. \right. \\
 & \quad \left. \left. (5.5689999999999995 \cdot 10^8 + \right. \right. \\
 & \quad \left. \left. 1.6000000000000008 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-2.73787477398843748 \cdot 10^{12} t} - \right. \right. \\
 & \quad \left. \left. 5.38900000000000023 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-1.36893738699421874 \cdot 10^{12} t} - \right. \right. \\
 & \quad \left. \left. 1.61670000000000024 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-1.94175515885704754 \cdot 10^{13} t} \right)^{1/2} \right) \\
 & \quad \left. \left((5.5689999999999995 \cdot 10^8 + \right. \right. \\
 & \quad \left. \left. 1.6000000000000008 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-2.73787477398843748 \cdot 10^{12} t} - \right. \right. \\
 & \quad \left. \left. 5.38900000000000023 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-1.36893738699421874 \cdot 10^{12} t} - \right. \right. \\
 & \quad \left. \left. 1.61670000000000024 \cdot 10^8 \right. \right. \\
 & \quad \left. \left. E^{-1.94175515885704754 \cdot 10^{13} t} \right)^{1/0.5} - \right. \\
 & \quad \left. 8.99670682899999896 \cdot 10^{-21} \right. \\
 & \quad \left. (5.5689999999999995 \cdot 10^8 + \right. \\
 & \quad \left. 1.6000000000000008 \cdot 10^8 \right. \\
 & \quad \left. E^{-2.73787477398843748 \cdot 10^{12} t} - \right. \\
 & \quad \left. 5.38900000000000023 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.36893738699421874 \cdot 10^{12} t} - \right. \\
 & \quad \left. 1.61670000000000024 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.94175515885704754 \cdot 10^{13} t} \right)^{1/2.5} + \\
 & \quad \left. 1.6155088557533147 \cdot 10^{-29} \right. \\
 & \quad \left. (5.5689999999999995 \cdot 10^8 + \right. \\
 & \quad \left. 1.6000000000000008 \cdot 10^8 \right. \\
 & \quad \left. E^{-2.73787477398843748 \cdot 10^{12} t} - \right. \\
 & \quad \left. 5.38900000000000023 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.36893738699421874 \cdot 10^{12} t} - \right. \\
 & \quad \left. 1.61670000000000024 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.94175515885704754 \cdot 10^{13} t} \right)^{1/3.5} \Big) \Big) / \\
 & \quad \left((5.5689999999999995 \cdot 10^8 + \right. \\
 & \quad \left. 1.6000000000000008 \cdot 10^8 \right. \\
 & \quad \left. E^{-2.73787477398843748 \cdot 10^{12} t} - \right. \\
 & \quad \left. 5.38900000000000023 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.36893738699421874 \cdot 10^{12} t} - \right. \\
 & \quad \left. 1.61670000000000024 \cdot 10^8 \right. \\
 & \quad \left. E^{-1.94175515885704754 \cdot 10^{13} t} \right)^{1/2} \Big) \Big) , \\
 & \quad \left\{ t, 4.500530182 \times 10^9 \times 365.2468, 4.500534082 \times 10^9 \times 365.2468 \right\} \Big]
 \end{aligned}$$

When the integration of deficit is carried out between 2000BC and 1900AD, Δ -T comes out to be: **53493.5217035582155** seconds.

It is the above integration carried out between the ancient time and 1900 which gives $\Delta - T$ for the given ancient time.

3. CALCULATION OF DELTA-T IN ANCIENT TIMES

The deficit to 86400 expression is integrated from 2000BC which corresponds to $4.500530182 \times 10^9 \times 365.2468$ days since the Giant Impact to 1900 of the current epoch which corresponds to $4.500534082 \times 10^9 \times 365.2468$ days since the Giant Impact. This integral works out to be 53493.5 seconds.

Using the above expression $\Delta - T$ is calculated from 2000BC to 2000AD and tabulated in Table (2.1).

Along with the calculated values the observed values are given alongside for comparison. Observed values are quoted from Stephenson & Houldon [1986]. This comprises data from 2000BC to 1600AD. These are pre-telescopic data and are based on ancient records of lunar and solar eclipse as preserved by the ancient civilizations. A more accurate set of data again based on the ancient records are quoted from Stephenson 1997 [1997]. This set of data from 500BC to 1600AD are based on the ancient records whereas from 1700AD to 2000AD are based on accurate telescopic observations.

Using the data tabulated in Table (2.1) graphical plots are generated from the calculated values, from the observed values as given by Stephenson & Houldon [1986] and from the observed values as given by Stephenson [1997]. The three plots are superimposed in the fourth plot.

Table 2.1 Calculated and observed Values of Delta-T

Time (t*) in Years since The Giant Impact	Time in year in the Current epoch	Calculated Values of Delta-T (seconds)	Observed Values of Delta-T in Telescopic era (secs.)	Values of Delta-T (St & H 1986) (seconds)	Values of Delta-T (Stephen 1997) (seconds)
4500534082	2000AD	-34.1657	64		
4500533982	1099AD	0	-2.7		
4500533882	1800AD	36.124	13.7		
4500533782	1700AD	142.538	9		
4500533682	1600AD	319.241		140	110
4500533582	1500AD	566.234		275	180
4500532382	1400AD	883.517		455	300
4500533382	1300AD	1271.09		680	470
4500533282	1200AD	1728.95		950	750
4500533182	1100AD	2257.1		1265	1100
4500533082	1000AD	2855.55		1625	1600
4500532982	900AD	3524.28		2035	2200
4500532882	800AD	4263.3		2531	3000
4500532782	700AD	5072.61		3120	3800
4500532682	600AD	5952.21		3802	4700
4500532582	500AD	6902.1		4577	5700
4500532482	400AD	7922.28		5445	6700
4500532382	300AD	9012.57		6406	7700
4500532282	200AD	10173.5		7461	8600
4500532182	100AD	11404.6		8608	9600
4500532082	000	12705.9		9848	10600
4500531982	100BC	14077.5		11181	11600
4500531882	200BC	15519.4		12607	12800
4500531782	300BC	17031.7		14126	14000
4500531682	400BC	18614.2		15738	15300
4500531582	500BC	20266.9		17444	16800
4500531482	600BC	21900		19242	
4500531382	700BC	23783.4		21133	
4500531282	800BC	25647		23117	
4500531182	900BC	27581		25194	
4500531082	1000BC	29585.2		27364	
4500530982	1100BC	31659.8		29627	
4500530882	1200BC	33804.6		31984	
4500530782	1300BC	36019.7		34433	
4500530682	1400BC	38205.1		36975	
4500530482	1500BC	40660.8		39610	
4500530482	1600BC	43086.7		42338	
4500530382	1700BC	45583		45159	
4500530282	1800BC	48149.5		48073	
4500530182	1900BC	50786.4		51081	
4500530082	2000BC	53493.5		54181	

Calculated values of Δ -T is plotted in Figure 2.1 by Mathematica Command.

```
q66 = ListPlot[{{(-2000, 53493.5), (-1900, 50786.4), (-1800, 48419.5),
  (-1700, 45583), (-1600, 43086.7), (-1500, 40660.8),
  (-1400, 38305.1), (-1300, 36019.7), (-1200, 33804.6),
  (-1100, 31659.8), (-1000, 29585.2), (-900, 27581),
  (-800, 25647), (-700, 23783.4), (-600, 21990), (-500, 20266.9),
  (-400, 18614.2), (-300, 17031.7), (-200, 15519.4),
  (-100, 14077.5), {0, 12705.9}, {100, 11404.6}, {200, 10173.5},
  {300, 9012.57}, {400, 7922.28}, {500, 6902.1}, {600, 5952.21},
  {700, 5072.61}, {800, 4263.3}, {900, 3524.28}, {1000, 2855.55},
  {1100, 2257.1}, {1200, 1728.95}, {1300, 1271.09},
  {1400, 883.517}, {1500, 566.234}, {1600, 319.241},
  {1700, 142.538}, {1800, 36.124}, {1900, 0}, {2000, -34.1657}}},
  PlotJoined → True, GridLines → Automatic,
  Frame → True, FrameLabel →  $\Delta$  - T (seconds),
  PlotLabel → {" $\Delta$ -T vs Years by Calculation"}]
```

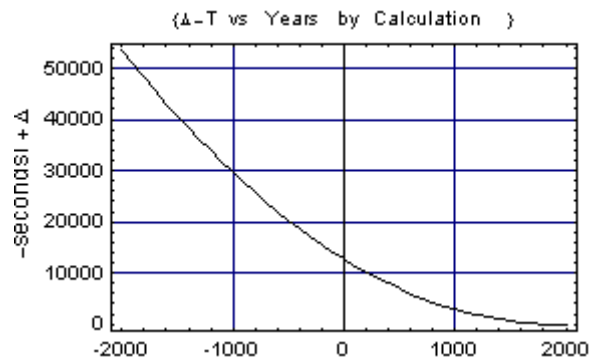


Figure.2.1: Theoretical Calculation of Δ -T vs Years

Observed values of Δ -T by Stephenson(1997) and by Stephenson&Houldon(1986) are plotted with time in Figure 2.2 and 2.3.

```
q77 = ListPlot[{{(-500, 16800), (-400, 15300),
  (-300, 14000), (-200, 12800), (-100, 11600), {0, 10600},
  {100, 9600}, {200, 8600}, {300, 7700}, {400, 6700},
  {500, 5700}, {600, 4700}, {700, 3800}, {800, 3000},
  {900, 2200}, {1000, 1600}, {1100, 1100}, {1200, 750},
  {1300, 470}, {1400, 300}, {1500, 180}, {1600, 110}, {1700, 9},
  {1800, 13.7}, {1900, -2.7}, {2000, 64}}}, PlotJoined → True,
  GridLines → Automatic, Frame → True, FrameLabel → (seconds),
  PlotLabel → {" $\Delta$ -T vs Years by Observation(St.1997)"}]
```

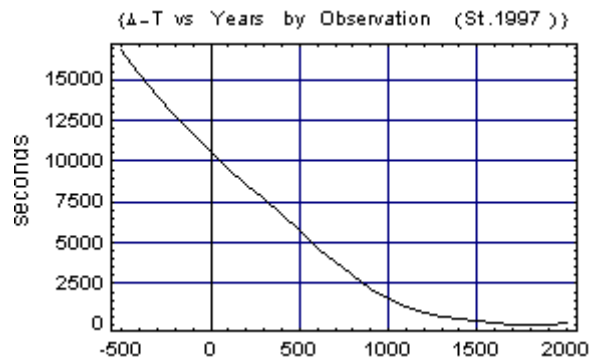


Figure 2.2: Plot of Observed Values(St.1997) of Δ -T with respect to time.

```
q88 = ListPlot[{{-2000, 54181}, {-1900, 51081}, {-1800, 48073},
  {-1700, 45159}, {-1600, 42338}, {-1500, 39610},
  {-1400, 36975}, {-1300, 34433}, {-1200, 31984},
  {-1100, 29627}, {-1000, 27364}, {-900, 25194}, {-800, 23117},
  {-700, 21133}, {-600, 19242}, {-500, 17444}, {-400, 15738},
  {-300, 14126}, {-200, 12607}, {-100, 11181}, {0, 9848},
  {100, 8608}, {200, 7461}, {300, 6406}, {400, 5445}, {500, 4577},
  {600, 3802}, {700, 3120}, {800, 2531}, {900, 2035},
  {1000, 1625}, {1100, 1265}, {1200, 950}, {1300, 680},
  {1400, 455}, {1500, 275}, {1600, 140}}, PlotJoined → True,
  GridLines → Automatic, Frame → True, FrameLabel → {seconds},
  PlotLabel → {"Δ-T vs Years by Observation(St.&H.1986)"]
```

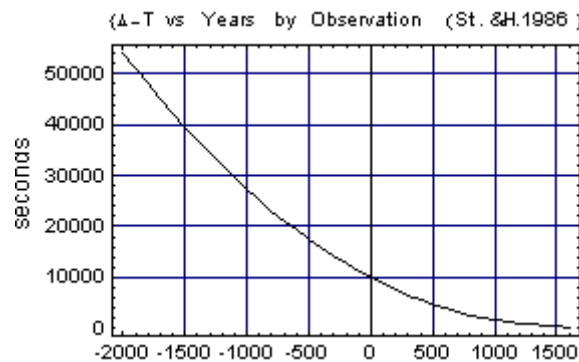


Figure 2.3: Plot of Observed Values(St.&Houlton 1986) of $\Delta-T$ with respect to time.

```
Show[{q66, q77, q88}, Axes → True]
```

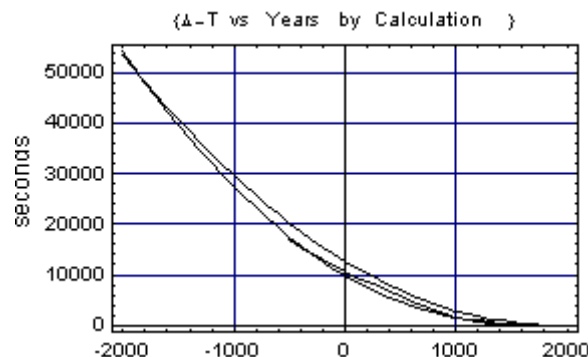


Figure 2.4: Superposition of Observed and Theoretical Curves.

4. DISCUSSION

As can be seen from the superposition of the three plots, the general trend is the same between the calculated values and the observed values except for the fact that calculated values are greater than the observed values throughout the epoch of observation except at the terminal points where they are coinciding. This is consistent with the actual geological happenings of the last 10,000 years.

In 8000BC the last Ice Age ended. The Polar Ice Caps which extend upto ± 75 degree latitude under warm periods extend upto ± 40 degree. Consequently there is an excessive thrust build-up on the polar ends of the Earth leading to a more oblate

shape during the Ice Age. As the polar Ice Caps melt and water is evenly distributed over the global surface area so is the excess thrust relieved with the thawing of Ice Age. This leads to Post-Glacial Rebound and a more spherical shape of the Earth. As the shape changes from oblate to sphere so does the Principal Moment of Inertia around the Polar axis decrease leading to an acceleration of the spin compensating partially the secular deceleration of the spin. The secular lengthening only due to Earth-Moon interaction is of the order of 1.0msec per century. But due to Post Glacial rebound it can become very small as seen in the observational data of Stephenson [1997]. It is primarily due to this that observed values of $\Delta - T$ are consistently smaller than the calculated values.

The calculation models only the Earth-Moon interaction whereas observed values have several components. There are irregular variations which average out in long term $\Delta - T$ but show up in short term $\Delta - T$.

There are periodic variations which also average out in long term estimate of $\Delta - T$ but do show up in short term estimates.

Then there are secular variations due to long term geophysical and geographical events. They will always show up both in long term as well as short term.

5. CONCLUSION

The general match of the calculated values with the observed values and the excess of the calculated over the observed value at the intermediate times is an excellent vindication of the Earth-Moon model presented in this series of papers. It does not only accurately model the basic Celestial-Mechanics but it gives a very important bench mark comparison which leads to the unravelling of factors, other than Earth-Moon gravitational effect, which are surreptitiously but surely affecting the period of rotation. By studying the residual variation in the period of rotation it is hoped that the Authors will be able to establish the nature of irregular variation-whether they are RANDOM or CHAOTIC. Once it is established that they are chaotic then the faint but definite precursors of the impending Earthquake and sudden volcanic eruptions are expected to be detected.

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Telescope. This Radio Telescope has enabled us to do World Class Research and make a place for India in the discovery of important celestial objects.

Last but not the least I express my gratitude to National Council of Science and Technology Network for bringing rural and urban middle school and high school children in ceaseless scientific quest and for organizing Children Science Congress. Today Children Science Congress, like Indian Science Congress, is involving 400 districts of 30 Indian states to take part in scientific research and enquiry.

Conflict of Interest:

There is no conflict of interest with anybody of any kind/

Appendix:

Definition:

Coordinated Universal Time (UTC): It is the Primary time standard globally used to regulate clock and time. It established a reference for the current time forming the basis of Civil Times and Time Zones. UTC facilitates communication, navigation, and scientific research. UTC time in India is Indian Standard Time (IST). IST is five and half hours ahead of UTC that is $IST = UTC + 5:30$. Greenwich Mean Time (GMT) is the local mean time at the Royal Observatory in Greenwich, London counted from midnight. From 1884 to 1972 GMT was the international standard of Civil Times. The Royal Observatory Greenwich is the home of GMT. A meridian is a North to South line used as a zero reference line for astronomical observations. Today GMT one day is reckoned from one mid-night to the following mid-night. The meridian in Greenwich is regarded as the Prime Meridian, which represents zero degree LONGITUDE. This line divides the eastern and western hemisphere. Historically anywhere on Earth longitude was measured in reference to the Prime Meridian of Greenwich. Historically the distance measured EAST or WEST from the Prime Meridian in Greenwich gave us the longitude. The Prime Meridian was defined by the position of the 'large transit' circle telescope at the Royal Observatory Greenwich. In 1767 the Fifth Astronomical Royal Nevil Maskelyne introduced the Nautical Almanac as part of the great 18th century quest to determine longitude. There were tables of lunar distances based on observations at Greenwich and using GMT as the time standard --this data enabled the navigators to find their positions at high sea. GMT was also crucial to the other great solution to the "longitude Problem" represented by John Harrison famous time keeper. British Mariners started keeping one chronometer set to GMT. This meant they could calculate their longitude from the Greenwich meridian which was 0 degree longitude by convention. The two solutions would help pave the way for GMT to become the world wide time standard a century later. 1850 and 1860 saw the expansion of railway and communication networks. At this juncture the need for National Standard of time arose. GMT became the Railway Time in England. Soon Great Britain adopted GMT as Britain's legal Time standard in 1880. By this time USA had chosen Greenwich as the basis of its time zones. In late 19th century 72% of World commerce depended on sea charts which used Greenwich as the prime meridian. So internationally Greenwich was accepted as zero degree longitude. The Airy Transit Circle (telescope) became the telescope that would define the Meridian of the World. Greenwich became the standard of the Universal Day. Shepherd Gate at the Royal Observatory

Greenwich has a clock which shows GMT to the public. It is a slave clock connected to a master clock. This system was installed in 1852. From 1852 to 1893 Shepherd Master Clock was at the heart of the British Time System. This standard time was sent by telegraph lines to all the cities of Great Britain. By 1866 time signals were sent to Cambridge University, Massachusetts, USA, via the new transatlantic submerged cables. The shepherd clock initially showed astronomical time starting at mid-noon. In 20th century counting was done from mid-night..

Terrestrial Time (TT) : International Astronomical Union has defined a time scale primarily for time measurements of astronomical observations from the surface of the Earth. It is defined by the orbital motion of Earth and other planets. It is the successor of Ephemeris Time (ET) based on System International SECOND. From 1952 to 1983 TT was known as ET. From 1984 to 2000 TT is known as Dynamical Time (TD). IERS (International Earth Rotation and Reference System Services) website gives full information on TT. Leap Second is added at the end of the day in certain given years and is denoted as 23:59:60 UTC. It is increasing and uniform time scale. This time scale is related to International Atomic Time (TAI) that in turn is defined by world wide array of interlinked atomic clocks. These clocks are based on International Standard (SI) SECOND. This SECOND was defined in 1967 by 13th General Conference on Weights and Measurements. Definition of SECOND is the duration of 9,192,631,770 periods of the electromagnetic radiation corresponding to the transit between the two hyper-fine levels of the ground state of Cesium133 atom.

Coordinated Universal Time (UTC) : UTC is based on slightly erratic spin period of Earth around its spin axis. It is formerly known as Greenwich Mean Time (GMT).

Delta T: TT-UT1 = Delta T This is a growing interval which has grown to little more than a minute. In precise time keeping as in Global Positioning System (GPS) the cumulative effect of the departure of Earth's spin period from the fixed length of day of International Atomic Time (TAI). ΔT has an effect on any computation involving topocentric observers but is particularly important in eclipses and occultation calculations.

Chaotic Variation in Spin Period of Earth: Earth's spin rate is slowing down and length of day (LOD) is lengthening. It has lengthened from 5 hrs to 24hrs in last 4.5Gy the life span of Earth. This is due to tidal braking of the spinning Earth. This tidal braking is causing 1.7 millisecond lengthening of day (LOD) per century at the present time. Earthquakes and sudden Volcanic eruptions also effect the spin rate. On 29th June 2022 LOD became 1.59ms shorter. Meltwater from the polar ice-caps combined with the shifting spin of Earth's core is effecting the spin of Earth in a way that negative leap seconds will have to be added.

The uneven Earth's spinning has been reported to affect geological processes, i.e. tectonism, seismicity and volcanism, on a planetary scale. Here, we show that changes of the length of day (LOD) influence eruptive activity at subduction margins. Statistical analysis indicates that eruptions with volcanic explosivity index (VEI) ≥ 3 alternate along oppositely directed subduction zones as a

function of whether the LOD increases or decreases. In particular, eruptions in volcanic arcs along contractional subduction zones, which are mostly E- or NE-directed, occur when LOD increases, whereas they are more frequent when LOD decreases along the opposite W- or SW-directed subduction zones that are rather characterized by upper plate extension and back-arc spreading. We find that the LOD variability determines a modulation of the horizontal shear stresses acting on the crust up to 0.4 MPa. An increase of the horizontal maximum stress in compressive regimes during LOD increment may favour the rupture of the magma feeder system wall rocks. Similarly, a decrease of the minimum horizontal stress in extensional settings during LOD lowering generates a larger differential stress, which may enhance failure of the magma-confining rocks. This asymmetric behaviour of magmatism sheds new light on the role of astronomical forces in the dynamics of the solid Earth. [Sotilli, Gianluca; Palladino, Danilo M.; Cuffaro, Marco; Doglioni, Carlo; (2015); "Earth's rotation variability triggers explosive eruptions in Subduction Zones", *Earth, Planets and Space*; **67**, Article Number 208 (2015);]

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