



Standard Geometric Brownian Motion for Short- Term Forecasting of Banks Stock Prices

Amadi, Innocent Uchenna¹ and Wokoma, Dagogo Allen²

Department of Mathematics/Statistics, Captain Elechi Amadi Polytechnic, Rumuola, Port Harcourt, Nigeria.

*Corresponding Author

Amadi, Innocent Uchenna

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Abstract: This system presents the classical Geometric Brownian Motion (GBM) benchmark with constant initial price. The Stochastic Differential Equation (SDE) assumes no deterministic trend, seasonality or regime effects. The simulation results for Fidelity Bank share prices were presented using a constant GBM model with varying sample paths. Four graphical solutions were analyzed to examine path behavior, expected growth, and sensitivity to drift and volatility. The results highlight the interaction between deterministic trend and stochastic noise in shaping Fidelity's price dynamics. The model serves as a benchmark for market efficiency with no deterministic trend. Also, the existence and uniqueness of the strong solution is proved under standard Lipschitz conditions. This system serves as the null model for comparing the other stochastic systems.

Keywords: Constant initial price, GBM, Share price, Sample paths, Drift, and Volatility.

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1.1 Introduction

Stock prices of commercial banks in Nigeria exhibit characteristics of randomness, volatility clustering, and drift, which makes deterministic forecasting inadequate. The Standard GBM, provides a parsimonious stochastic framework to capture these features for short-term horizons[1] and [2] respectively. Under GBM, the log-returns of a bank's share price are assumed to be normally distributed and independent, leading to the Stochastic Differential Equation (SDE). The closed-form solution allows for direct simulation and probabilistic forecasting over days to weeks[3]. For banks such as Access Bank and Fidelity Bank, short-term forecasting is critical for trading desks, risk managers and retail investors who need to position around earnings announcements, dividend dates, and Central Bank of Nigeria (CBN) policy releases. By calibrating drift and volatility from recent historical data, the standard GBM can generate confidence bands for future prices and quantify Value-at-Risk (VaR) [4]. Thus, this study

adopts standard GBM as a baseline model for short-term forecasting of bank stock prices. It provides a benchmark against which more complex models with seasonality, regime-switching, and correlation can be compared [2].

At the same time, the stock market is typically avowed for its high returns. Returns on investment for some capital market are approaches for aptly evaluating returns on an investment. Investors adopt these measures to evaluate or assess the significance of expenditure [5]. In the capital market, the fluctuations of stock prices are caused by changes of their values over time.

Though, [6] looked at the stochastic model of price changes on the floor of stock market. Thus, the equilibrium price and the market growth rate of assets were found. [7] studied the stochastic analysis of stock market expected returns and growth-rates and, the growth-rates of four different assets were considered. In the same manner, [8] studied the stochastic model of the changes of stock market prices. More so, [9] studied the unstable nature of stock market

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forces using proposed differential equation model. In the work of [10] stochastic analysis of the behavior of stock prices were examined and results showed that the proposed model is efficient for the prediction of stock prices. [11] worked on the solution of differential equations and stochastic differential equations of time-varying investment returns; where precise conditions were obtained which governs stock return rates through multiplicative effect and multiplicative inverse trends series. [12] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this study, the drift and volatility coefficients for the stochastic differential equations were determined. [13] examined the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test were studied, [14] considered the stochastic analysis of Markov Chain in Finite state. The transition Matrix replicated the use of 3-State transition probability matrix which enables them to present precise condition of obtaining expected mean rate of returns of each stock.

On the hand, investors often over fit trends to random walks. Most Nigerian studies do not test the null hypothesis of "no deterministic component" before adding complexity. This paper aims to establish a benchmark GBM model for Fidelity Bank and evaluate its suitability for short-term risk assessment. Hence, this paper compliments the works of [15] and [16] respectively by considering lack of a baseline GBM to compare against seasonal and trend models for Nigerian Stock Exchange (NSE) stocks.

The arrangement of this paper is set as follows: Section 2.1 presents the mathematical framework, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1: Mathematical Framework

At this stage we present few intricate definitions touching this dynamic area of study, hence we have the following:

Definition 1: If $\mathcal{F}$ is a σ -algebra in Ω , then Ω is called a measurable space and the members of $\mathcal{F}$ are called the measurable sets in Ω .

Definition 2: Let $(\Omega, \mathcal{M})$ be a measurable space. A map $\mu: \mathcal{M} \rightarrow \mathbb{R}^+ = [0, \infty) \cup \{\infty\}$ is called a measure provided that

$$(i) \quad \mu(\emptyset) = 0$$

$$(ii) \quad \mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$$

Definition 3: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T , $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 4: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Where X_k are independent and identically distributed random variables. Therefore μ is not known completely which is subject to environmental effects. Therefore the stock dynamics governing this process can be written as :

$$dS(t) = \mu S(t)dt + \sigma S(t)dZ(t)$$

Where σ is the volatility, dZ is the Brownian motion .

Definition 5: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space with filtration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), \dots, W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g as follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t), 0 \leq t \leq T,$$

$$X(0) = x_0,$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f: [0, T] \times \mathbb{R}^n$ and $g: [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$.

Theorem 1.1: (Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathbb{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higher order terms (h.o.t)}$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$\begin{aligned} dF_t &= \frac{\partial F}{\partial S_t} (a_t dt + b dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (a_t dt + b dz(t))^2 \\ &= \frac{\partial F}{\partial S_t} (a_t dt + b dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b^2 dt, \\ &= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b^2 \right) dt + \frac{\partial F}{\partial S_t} b_t dz(t) \end{aligned}$$

More so, given the variable $S(t)$ denotes stock price, then following GBM implies, the function $F(S, t)$, Ito's theorem gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

[17-22].

2.1.1 Problem Formulation

We consider Geometric Brownian Motion models for Nigerian bank stock prices under different initial condition structures. Here, we consider single Asset constant Geometric Brownian Motion (GBM) with constant Initial condition. Baseline model for Fidelity Bank price under standard GBM assumption with no deterministic trend in the initial condition. Therefore, we model the stock.

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t, & t > 0 \\ S_0 = \phi_t \end{cases} \quad (1.1)$$

where S_0 represents initial price, ϕ_t represent constant term, μ represents drift coefficient, σ represents volatility coefficient, S_{BASE} is the base price level, A is seasonal amplitude, ω is seasonal frequency, W_t .

2.1.2 Method of Solution

We apply Theorem 1.1 in solving (1.1)-(1.3) which consist of a system of variable coefficient problem whose solutions are not trivial. let $f(S_t, t) = \ln S_t$ and taking the partial derivatives yields:

$$\left. \frac{\partial f}{\partial S_t} = \frac{1}{S_t}, \frac{\partial^2 f}{\partial S_t^2} = -\frac{1}{S_t^2}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.2)$$

According to Ito's theorem gives:

$$df(S_t, t) = \sigma S_t \frac{\partial f}{\partial S_t} dW_t + \left(\mu S_t \frac{\partial f}{\partial S_t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} + \frac{\partial f}{\partial t} \right) dt \quad (1.3)$$

Substituting (1.2) into (1.3) gives the following:

$$df(S_t, t) = \sigma S_t \frac{1}{S_t} dW_t + \left(\mu S_t \frac{1}{S_t} + \frac{1}{2} \sigma^2 S_t^2 \left(-\frac{1}{S_t^2} \right) + 0 \right) dt \quad (1.4)$$

simplifying gives

$$\begin{aligned} &= \sigma S_t \frac{S_t}{S_t} dW_t + \left(\mu \frac{S_t}{S_t} - \frac{1}{2} \sigma^2 \frac{S_t^2}{S_t^2} \right) dt \\ &= \sigma S_t dW_t + \left(\mu - \frac{1}{2} \sigma^2 \right) dt \end{aligned} \quad (1.5)$$

$$df(S_t, t) = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \quad (1.6)$$

Integrating (1.6) from 0 to t gives

$$\int_0^t d \ln S_t = \int_0^t df(S_t, u) = \int_0^t \left(\mu - \frac{1}{2} \sigma^2 \right) du + \int_0^t \sigma dW_t du \quad (1.7)$$

$$\ln S_t - \ln S_0 = \left(\mu u - \frac{1}{2} \sigma^2 u \right)_0^t + (\sigma dW_t u)_0^t$$

$$\ln \left(\frac{S_t}{S_0} \right) = \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t$$

Taking the ln of the both sides gives

$$S_t = S_0 \exp \left(\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) \quad (1.8)$$

This gives the general solution of the SDE. To find the particular solution

$$S_t(0) = \phi_t, \Rightarrow S_0 \exp \left(\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) = \phi_t$$

$$S_0 = \phi_t$$

$$S_t = \phi_t \exp \left(\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma dW_t \right) \quad (1.9)$$

2.1.3 Constant GBM Fidelity Bank Assumptions

1. $S_t > 0$ a.s. a.s.
2. The $\mu, \sigma > 0$ positive constants.
3. The $S(0) = S_0 > 0$ is deterministic
4. $\{W_t\}$ is standard Brownian Motion.

Theorem (1.2): Existence and Uniqueness

Following 1-4 assumptions, we have :

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t, & t > 0 \\ S_0 = \phi_t \end{cases}$$

has unique strong solution:

$$S_t = \phi_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dW_t\right)$$

Proof: Identical

let $X_t = \ln(S_t)$.By Ito's theorem

$$\begin{aligned} dX_t &= \sigma S_t \frac{S_t}{S_t} dW_t + \left(\mu \frac{S_t}{S_t} - \frac{1}{2}\sigma^2 \frac{S_t^2}{S_t^2}\right)dt = \sigma S_t dW_t + \left(\mu - \frac{1}{2}\sigma^2\right)dt \\ &= \sigma S_t dW_t + \left(\mu - \frac{1}{2}\sigma^2\right)dt = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t \end{aligned}$$

Integrating 0 to t gives:

$$X_t = X_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dW_t$$

Exponentiating and using

$S_0 = S_0(t)$ gives the claimed solution. For existence and

uniqueness: Define $a(t, x) = \mu$ and $b(t, x) = \sigma$. Then

$$|a(t, x) - a(t, y)| + |b(t, x) - b(t, y)| \leq (\mu + \sigma)|x - y|$$

, the coefficient satisfy Lipschitz and linear growth holds. So standard SDE theorem applies.

3.1 Results and Discussion

The following parameter values were explicitly used $S_0 = 15, \mu = 0.10, \sigma = 0.35$.

The single-asset baseline model:

$$S_t = \phi_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dW_t\right).$$

This is the exact analytical solution of

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t, & t > 0 \\ S_0 = \phi_t \end{cases}$$

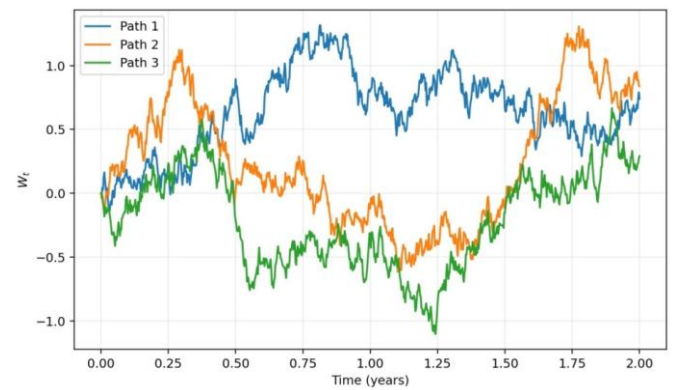


Figure 1: Three Brownian-motion paths for Model .

The three Brownian paths and corresponding exact GBM trajectories are displayed in Figures 1 and 2. Because the price is obtained through an exponential transformation, sufficiently positive Brownian realizations can generate rapid increases, while sufficiently negative realizations can generate substantial declines. Nevertheless, the price remains positive for all finite t . More so, the three simulated paths for Fidelity share price originate from the same initial value, indicating a common starting valuation. Initially, the paths remain closely aligned, reflecting minimal impact of stochastic shocks. Subsequently, path 1 trends upward while path 2 and 3 trend downward due to the Brownian component. However, in the later period all paths realign upward, which suggests that the positive drift ultimately dominates and restores the long-term tendency of Fidelity despite short-term divergence.

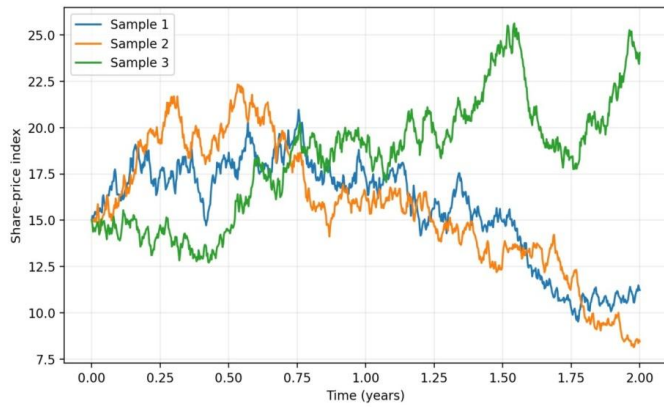


Figure 2: Three analytical GBM paths for Model

The three paths for Fidelity Bank begin at a common point but immediately diverge, illustrating the early influence of randomness in price formation. After a period of separation, the paths converge toward a central level, indicating the stabilizing effect of drift term in the GBM. Thereafter, path 3 moves upward while path1 and 2 move downward, demonstrating how idiosyncratic shocks produce different outcomes. Overall, the Figure 2 shows that Fidelity's share price is governed by both a drift-driven central tendency and volatility-driven short-term fluctuations.

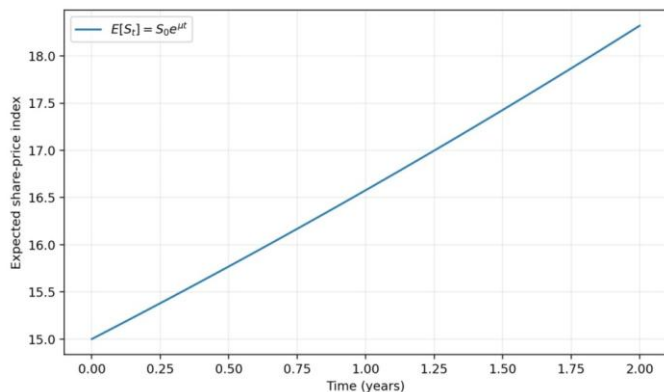


Figure 3: Expected value $E[S_t] = S_0 e^{\mu t}$ for Model .

Figure 3 therefore shows a smooth exponential expected-value curve. The expected value depends on μ but not directly on σ . Volatility affects higher moments and the distribution of outcomes rather than the GBM mean. For a GBM, the expected price is $E[S_t] = S_0 e^{\mu t}$. The derivation follows from the moment-generating property of the Gaussian variable $W_t \approx N(0, t)$ $W_t \sim N(0, t)$. In this circumstance, the expected value is represented by a positive straight line, indicating exponential growth in Fidelity's average share price over time. This result follows directly from the constant GBM assumption where the deterministic drift term dominates the long-run behavior. Although individual sample paths may deviate significantly

due to volatility, the expected trajectory confirms a positive long-term outlook for Fidelity under the model.

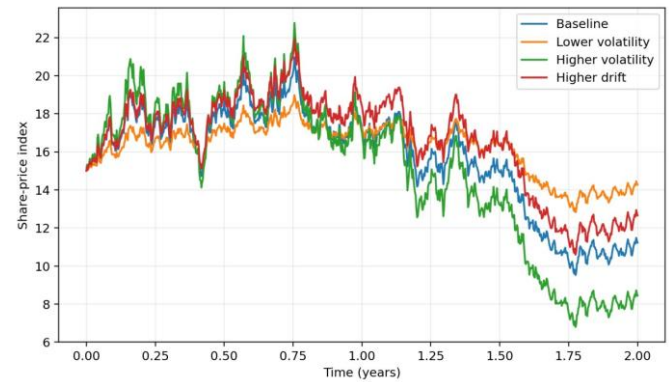


Figure 4: Sensitivity of the constant GBM model to drift and volatility.

The sensitivity plot in Figure 4 shows that larger drift shifts the growth tendency upward, whereas larger volatility increases the path's stochastic departure from its central tendency. The effect of volatility should therefore be interpreted probabilistically rather than as a deterministic upward or downward adjustment. However, the sensitivity plot illustrates the effect of varying drift and volatility on Fidelity's share price dynamics. An increase in drift shifts the central tendency upward, thereby raising the expected growth rate of the share price. In contrast, an increase in volatility widens the dispersion of sample paths around the central tendency, reflecting greater uncertainty. The result implies that drift determines Fidelity's long-term direction while volatility governs short-term risk and variability.

4.1 Conclusion

The existence and uniqueness result validates the benchmark as a well-defined stochastic process. The model exhibits wide path dispersion and market efficiency. All risk is idiosyncratic, making technical signals unreliable. The conclusion supports passive investment and stop-loss risk control. This paper confirms that any extension to non-constant $S_0(t)$ must be statistically justified, providing the baseline for testing seasonality, growth, and regime effects. In all, (i) the analysis shows that Fidelity's long-term share price is driven by positive drift, while volatility causes short-term deviations. (ii) sample paths confirms that despite initial divergence, the central tendency reasserts itself over time (iii) the risk management for Fidelity should focus on volatility control, while investment decisions can rely on the upward drift for long-horizon positioning. More so, modification of the proposed model will be an interesting study to explore.

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