

Time-varying conditional copula modelling of dependence structure in Indian financial markets: An empirical analysis

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Abstract: This paper proposes skew-t copula with a time varying conditional correlation model to capture the time varying symmetric heavy-tail dependence structure among multi-asset classes, including government bonds, corporate bonds, equities, and real estate investment trusts (REITs). We provide new evidence that lower the dependence coefficients reflecting changes in the market volatility while maintaining symmetry between the upper and lower tails. The degree of joint heavy-tailed dependence among asset classes significantly increased and leading to a decline in the diversification benefit of multi-asset portfolios. Our empirical analysis shows that in terms of AIC and BIC, symmetric skew-t copula fits data of multi-asset classes better than to conventional time varying elliptical copulas and static t-copulas. Furthermore, out-of-sample analysis shows that considering a symmetry of heavy-tail behavior improves the expected shortfall (ES) estimation accuracy and enhances the performance of minimum-ES portfolios. The findings emphasize that capturing time-varying symmetric heavy-tail dependence is essential for accurate risk assessment and effective portfolio management.

Keywords: Investment, portfolio, skew-t copula, tail dependence and heteroskedasticity.

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1. Introduction

The modeling of dependence among financial assets has seen substantial progress since the introduction of copula theory in statistics (Sklar, 1959). Copulas allow the separation of marginal behavior from joint dependence structure, permitting great flexibility in multivariate modeling (Embrechts et al., 2003) and (Joe, 2014). The survey by Aas et al., (2016) highlights how pair-copula constructions (vines) extended multivariate modeling beyond simplistic, rigid copulas by composing bivariate building blocks. More recently, (Ismail et al., 2023) provide a comprehensive review of copula developments in finance, emphasizing the increasing use of asymmetry and time-variation in dependence. because financial returns often display tail dependence even when linear correlation is weak, copula models capable of capturing extreme co-movements have become central in risk modeling (McNeil et al., 2005). The concept of tail dependence (upper and lower) formalizes the probability of simultaneous extreme events in joint tails. Static copulas (Gaussian, Student-t, Clayton, Gumbel) differ in their tail

dependence properties (Clayton has lower-tail dependence, Gumbel has upper-tail dependence), but many are symmetric or impose restrictive structures (Patton, 2006; Joe, 2014).

However, static copulas may not adequately represent time-variation, especially under stress periods in financial markets. For that reason, time varying copula models where the dependence parameters evolve over time have become popular. The author Patton, (2006, 2012) introduced conditional copulas in time series, and many extensions (for example, time varying t-copulas based copulas, time-varying mixture copulas). The work of (Cortese et al., 2019) investigates tail correlation over time among equity indices using time varying copulas, demonstrating improved value-at-risk (VaR) forecasts versus static correlation models.

Time varying copula models are particularly advantageous for capturing symmetric tail dependence, i.e. situations where the strength of dependence differs between joint downside events versus joint upside events. For instance, in crises, asset classes often co-move more strongly downward, a phenomenon that symmetric models cannot capture. Models such as the symmetrized

Joe–Clayton (SJC) copula allow separate upper and lower tail dependence parameters (Joseph, 2020; Patton, 2006). The authors Deng et al., (2023) develop large skew-t copula models that accommodate symmetric tail dependence in high-dimensional equity returns, and their Bayesian variational inference approach exploits latent factor structure. They extend the class of skew-t copulas to large portfolios, capturing heterogeneity in symmetric dependence across pairs. Meanwhile, the Ito et al., (2025) propose a time varying symmetric copula modelling multi-asset dependence, blending autoregressive time varying with skew-t copula structure. Parallel to parametric approaches, semi or nonparametric time varying copula models have been developed to mitigate model misspecification. The author Pareek and Ghosh, (2025) design a semiparametric time varying copula framework combining flexible empirical copulas with parametric margins, applied to U.S., India, and Hong Kong returns. Also, factor latent dependent copula models (Joe, 2024) and factor–copula approaches allow richer structure and extreme-value inference enhancements in high dimensions.

The use of copulas in finance is now well-established. The monograph by Breymann et al., (2003) on “Copula risk models” remains a foundational reference, and the review by Embrechts et al., (2003) discuss inference, goodness-of-fit, and simulation in financial applications. The Sahamkhadam, (2021) explores copula-based portfolio optimization, studying how different copula families influence tail risk and wealth outcomes. In empirical settings, copula-based models have been used to examine extreme spillovers (Dai & Zhou, 2023) on commodity futures–spot markets, cross-market dependence (Albulescu et al., 2020) among energy or agriculture or metal (Alqaralleh, 2019) for volatility connectedness and regime-switching dependence (Rehman et al., 2023). The literature also shows comparisons between time varying correlation models and copula models (Cortese et al., 2019) and show copula models often outperform dynamic normal copula and dynamic t-copula (DNC and DtC) in tail risk forecasting.

The explicit modeling of time-varying symmetric dependence among Indian asset classes (equity, bonds, REITs) remains underexplored. We proposed a dynamic conditional skew copula and applied it to Indian asset classes, integrated macro covariates, and delivered empirical insight into tail dependence under market crisis.

2. Methodology

In this section, we first provide a brief introduction to Gaussian copula theory. Then, we illustrate the marginal model and time varying Gaussian copulas including DNC and DtC. The dynamic gaussian heterogeneity skew-t copula (DGHSktC) and dynamic conditional skew-t copula (DCSktC) are our proposed models. The back-testing, such as portfolio optimization is carried out using these models.

2.1. Copula based dependences modelling

A Gaussian copula is an actuarial function that analyze the dependence structure between random variables separately from their marginal behavior. The fundamental idea originates from Sklar’s theorem, which states that any multivariate joint distribution can be decomposed into its marginal distributions and copulas them together. A random vector $\underline{X} = (X_1, X_2, \dots, X_n)$ with

a continuous marginal cumulative distribution function (CDFs) $F_1, F_2, \dots, F_n$. The joint CDF H can be expressed as:

$$H(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n); \Theta) \quad (1)$$

where, $C(F_1(x_1), F_2(x_2), \dots, F_n(x_n); \Theta)$ is the Gaussian copula parameter and $C: [0,1]^n \rightarrow [0,1]$. Let $x = (x_1, x_2, \dots, x_n)'$ and $u = (u_1, u_2, \dots, u_n)'$ with $u_i = F_i(x_i)$. The joint density function L can be written as

$$L = c(u) \prod_{i=1}^n f_i(x_i) \quad (2)$$

Where f_i are marginal densities and $c(u) = \frac{\partial^n C(u)}{\partial u_1 \dots \partial u_n}$ is the copula density. The Gaussian copula is defined by applying the multivariate standard normal CDF to transformed uniform marginals. Let Φ be the standard normal CDF and Φ_n is the n-variate normal CDF with correlation matrix Σ . The Gaussian copula $C_N(u; \Sigma)$ is

$$C_N(u; \Sigma) = \Phi_n(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \Sigma) \quad (3)$$

$$C_N(u; \Sigma) = \frac{1}{\sqrt{|\Sigma|}} \exp \left\{ -\frac{1}{2} x' (\Sigma^{-1} - I) x \right\}, \text{ with } x = \Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n) \quad (4)$$

2.2. Marginal model specification (ARMA-GARCH, Skew-t)

We address the properties of volatility clustering, leptokurtosis and conditional heteroscedasticity from ARMA-GARCH family models. These standardized residuals serve as inputs for copula estimation. The ARMA-GARCH Models specification is the continuous compound return of asset i at time t : $r_{it} = \ln \left(\frac{P_{it}}{P_{i,t-1}} \right)$, where P_{it} is the price of asset i at time t . The return process is modeled as: $r_{it} = \mu_{it} + \epsilon_{it}$, where μ_{it} represents the conditional mean and ϵ_{it} is the innovation term with conditional variance of equation (8). We define the conditional mean equation of ARMA(p, q). Mean of ARMA (1) and Volatility of GARCH (1,1) of standardized residuals following Hansen’s skew-t, where, $z_{i,t} \sim Skt(0,1, \lambda_i, \gamma_i)$.

Model each return $r_{i,t}$ as:

$$r_{i,t} = \mu_i + \phi_i r_{i,t-1} + \theta_i \epsilon_{i,t-1} + \epsilon_{i,t}, \quad (5)$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i I_{\{\epsilon_{i,t-1} < 0\}} \epsilon_{i,t-1}^2 \quad (6)$$

$$\mu_{it} = \phi_0 + \sum_{j=1}^p \phi_j r_{i,t-j} + \sum_{k=1}^q \theta_k \epsilon_{i,t-k} \quad (7)$$

where, ϕ_j : autoregressive coefficients and θ_k : moving average coefficients.

The conditional variance equation GARCH(m, n) is given as

$$h_{i,t} = \omega + \sum_{j=1}^m \alpha_j \epsilon_{i,t-j}^2 + \sum_{k=1}^n \beta_k h_{i,t-k} \quad (8)$$

where, $h_{i,t}$: conditional variance at time t , $\omega > 0$: long-term variance component, α_j : short-term shock effects (ARCH terms) and β_k : persistence of volatility (GARCH terms). The GARCH(1,1) model captures volatility persistence efficiently given as

$$h_{i,t} = \omega + \alpha_1 \epsilon_{i,t-1}^2 + \beta_1 h_{i,t-1} \quad (9)$$

Once the ARMA-GARCH model is estimated by the standardized residuals and are obtained as: $z_{it} = \frac{\epsilon_{it}}{\sqrt{h_{it}}}$ where, z_{it} should be approximately i.i.d. with zero mean and unit variance. These standardized residuals are transformed into uniform [0,1] variables using the empirical cumulative distribution function (CDF): $u_{it} = \hat{F}_i(z_{it})$.

2.3. Proposed Dynamic copula model for dependence structure among multi-asset classes

The dynamic symmetric conditional Skew-t copula (DSCSkTC) modeled as:

$$C_t(u_1, \dots, u_n; \theta_t) = CSkt(u_1, \dots, u_n; \rho_t, \lambda_t, \nu) \quad (10)$$

$$\rho_t = (1 - \alpha - \beta)\bar{\rho} + \alpha z_{t-1} z_{t-1}' + \beta \rho_{t-1} \quad (11)$$

The proposed dynamic symmetric conditional symmetric skew-t copula (DSCSkTC) is:

$$C_t(u_1, \dots, u_n) = C_{Sym-Skt}(u_1, \dots, u_n; \theta_t) \quad (12)$$

$$\theta_t = \omega + Af(z_{t-1}) + B\theta_{t-1} + \Gamma X_t \quad (13)$$

It captures the both symmetric dependence persistence and covariate driven shifts. Let $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$ be the vector of standardized marginal innovations from ARMA-GARCH model filtering. The model reveals the time-varying, symmetric heavy-tail dependence through a symmetric skew-t copula. x_{it} be the inverse-transformed vector, $F_i(\cdot)$ and $f_i(\cdot)$ are marginal CDF and PDF of the i^{th} symmetric skew-t copula construction. The PDF of symmetric skew-t copula model is given as

$$f_x(x_t; \Sigma, \gamma, k) = \frac{\Gamma(\frac{\gamma+n}{2})}{\Gamma(\frac{\gamma}{2})^{(n/2)} |x_t|^{\frac{\gamma}{2}}} \left(1 + \frac{x_t' \Sigma^{-1} x_t}{\gamma}\right)^{-\frac{\gamma+n}{2}} \left[1 + k \frac{(x_t' \Sigma^{-1} x_t)^2}{x_t' \Sigma^{-1} x_t + 1}\right] \quad (14)$$

If $\gamma > 0$ it controls tail thickness and $k \in [-1, 1]$ is a symmetric skew-intensity parameter.

The CDF of symmetric skew-t copula model.

$$F_x(x_t; \Sigma, \gamma, k) = \int_{-\infty}^{x_t} \left(\frac{\Gamma(\frac{\gamma+n}{2})}{\Gamma(\frac{\gamma}{2})^{(n/2)} |x_t|^{\frac{\gamma}{2}}} \left(1 + \frac{x_t' \Sigma^{-1} x_t}{\gamma}\right)^{-\frac{\gamma+n}{2}} \left[1 + k \frac{(x_t' \Sigma^{-1} x_t)^2}{x_t' \Sigma^{-1} x_t + 1}\right] \right) dx_t \quad (15)$$

where $\epsilon_{t,i}$ is assumed to be uncorrelated with $r_{s,i}$ for $s < t$, and $K_i + O_i \leq 1$ for variance stationarity. In (5) and (7), μ_i is the unconditional mean parameter and ρ_i is the AR(1) parameter. In addition, ω in (6) is the unconditional variance parameter, and K_i and O_i are for the GARCH effect. We assume the AR(1) for returns to avoid model complexity and to consider whether returns of each asset class exhibit a momentum effect or reversal effect in empirical analysis. We adopt GARCH(1,1) to avoid model complexity, and as described in (Hansen, 1994). Following Christoffersen et al., (2018), and Zhou and Qin, (2021), we assume a standardized skew-t distribution from Hansen, (1994), for the i.i.d. standardized return residuals, which is given by

$$skewt(\tilde{S}; \gamma, t) = bc \left[1 + \frac{1}{\gamma - 2} \left(\frac{b\tilde{S} + a}{1 - t} \right)^2 \right]^{-\frac{t+1}{2}} \text{ if } \tilde{S} \geq -\frac{a}{b}$$

$$a = 4tc \left(\frac{\gamma-2}{\gamma-1} \right), \quad b = \sqrt{1 + 3t^2 - a^2}, \quad c = \frac{\Gamma(\frac{\gamma+1}{2})}{\sqrt{\pi(\gamma-2)}\Gamma(\frac{\gamma}{2})}$$

where, $\gamma \in (2, \infty)$ is the degree of freedom, and $l \in (-1, 1)$ is the skewness parameter. As described in Demarta and McNeil, (2005), the t-copula density function c^t is given by

$$c^t(u; \Sigma, \gamma) = \frac{\Gamma(\frac{\gamma+n}{2}) \prod_{i=1}^n \left(1 + \frac{1}{\gamma} x_i^2\right)^{\frac{\gamma+1}{2}}}{\sqrt{|\Sigma|} \Gamma(\frac{\gamma}{2})^{1-n} \Gamma(\frac{\gamma+n}{2})^n + \left(1 + \frac{x_t' \Sigma^{-1} x_t}{\gamma}\right)^{\frac{\gamma+n}{2}}} \quad (16)$$

where, Σ is the correlation matrix, γ is the degree of freedom parameter, $\Gamma(\cdot)$ is the gamma function, $u = (u_1, \dots, u_n)'$, $u_i \in [0, 1]$ for $i = 1, \dots, n$, and $x = (x_1, \dots, x_n)'$ with $x_i = T_1^{-1}(u_i; \gamma)$ being quantile function of the standard t distribution. The Gaussian heterogeneity symmetric skew-t copula (GHSktC) can capture linear and symmetric tail dependence structures,

$$c^{GHSktC}(u; \Sigma, \nu, \gamma) = \frac{2^{\frac{(\nu-2)(n-1)}{2}} K_{\nu+n} \left(\sqrt{\gamma' \Sigma^{-1} \gamma} (\gamma + x' \Sigma^{-1} x) \right)^{\frac{\gamma+n+1}{2}} e^{x' \Sigma^{-1} x}}{\sqrt{|\Sigma|} \Gamma(\frac{\gamma}{2})^{1-n} \left(1 + \frac{x_t' \Sigma^{-1} x_t}{\gamma}\right)^{\frac{\gamma+n}{2}}} \prod_{i=1}^n \frac{\left(|y_i| \sqrt{\gamma + x_i^2} \right)^{-\frac{\gamma+1}{2}} \left(1 + \frac{1}{\gamma} x_i^2\right)^{\frac{\gamma+1}{2}}}{K_{\frac{\gamma+n}{2}} \left(|y_i| \sqrt{\gamma + x_i^2} \right) e^{x_i^2 / \gamma}} \quad (17)$$

The conditional symmetric skew-t copula (CSktC) density function c^{CSktC} is given by

$$c^{CSktC}(u; \Sigma, \nu, \gamma) = 2^{n-1} \frac{t_n(x; \Omega, \gamma) T_1 \left(\tilde{a}' x \sqrt{\frac{\gamma+n}{x' \Omega^{-1} x + \gamma}} (\gamma + n) \right)}{\prod_{i=1}^n t_1(x_i; \gamma) T_1(x_i; \gamma + 1)} \quad (18)$$

Where, $\tilde{a} = \frac{\Omega^{-1} \delta}{\sqrt{1 - \delta' \Omega^{-1} \delta}}$, $\Omega = \Delta(\Sigma + \xi \xi') \Delta'$, $\Delta = \text{diag}(\sqrt{1 - \delta_1^2}, \dots, \sqrt{1 - \delta_n^2})$ and $\xi = \left(\frac{\delta}{\sqrt{1 - \delta^2}} \right)$. Ω is the scale matrix and δ is the vector of skewness parameter. $t_n(x; \Omega, \gamma)$ is the PDF of n -variate t distribution with the correlation matrix Ω and the degrees of freedom γ , $T_1(x_i; \gamma + 1)$ is the CDF of univariate t distribution with the degrees of freedom γ . In this paper, we introduce conditional time varying copula into the correlation matrix of each copula and utilize it as a dynamic copula to capture the dynamic dependence structure between variables.

2.4. Time varying Dependences

The dynamic process of time-varying dependencies is given by

$$\Sigma_t = \text{diag}(Q_t)^{-\frac{1}{2}} Q_t \text{diag}(Q_t)^{-\frac{1}{2}} \quad (19)$$

$$Q_t = Q(1 - \alpha - \beta) + \alpha \Psi(x_{t-1}, Q_{t-1}) + \beta Q_{t-1} \quad (20)$$

Where, Q is an unconditional correlation matrix, x_t is a vector of the quantiles of the univariate standard distribution for each copula at time t , α and β satisfy $\alpha, \beta > 0, \alpha + \beta < \infty$. From the assumption of conditional time varying dependence,

$$E[Q_t] = Q \quad (21)$$

It is necessary to give $\Psi(x_{t-1}, Q_{t-1})$ based on each time varying copula considering the moments of the involved distribution in each copula to satisfy equation (21). Here, we introduce time varying GHSktC.

$$\Psi^{DGHSkt}(x_{t-1}, Q_{t-1}) = \text{diag}(Q_t)^{-\frac{1}{2}} \eta^{DGHSkt}(x_{t-1}) \text{diag}(Q_t)^{-\frac{1}{2}} \quad (22)$$

$$\text{where, } \eta^{DGHSkt} = \frac{\gamma-2}{\gamma} \left(\left(x_t - \frac{\gamma \gamma'}{\gamma-2} \right) \left(x_t - \frac{\gamma \gamma'}{\gamma-2} \right)' \left(\frac{2\gamma^2 \gamma \gamma'}{(\gamma-2)^2 (\gamma-4)} \right) \right) \quad (23)$$

$$\Psi^{DGHSkt}(x_{t-1}, Q_{t-1}) = \text{diag}(Q_t)^{-\frac{1}{2}} (\Delta^{-1} \Omega_{t-1} (\Delta^{-1})' - \xi \xi') \text{diag}(Q_t)^{-\frac{1}{2}} \quad (24)$$

$$\text{where, } \Omega_t = \frac{\gamma-2}{\gamma} \left(\eta^{DSCskTC}(x_t) + \frac{\gamma}{\pi} \left(\frac{\Gamma(\frac{\gamma-1}{2})}{\Gamma(\frac{\gamma}{2})} \right)^2 \delta \delta' \right)$$

$$\eta^{DGHsktC}(x_t) = \left(x_t - \sqrt{\frac{\gamma}{\pi}} \frac{\Gamma\left(\frac{\gamma-1}{2}\right)}{\Gamma\left(\frac{\gamma}{2}\right)} \delta \right) \left(x_t - \sqrt{\frac{\gamma}{\pi}} \frac{\Gamma\left(\frac{\gamma-1}{2}\right)}{\Gamma\left(\frac{\gamma}{2}\right)} \delta \right)' \quad (25)$$

2.5. Estimation procedure

We derive the estimating equations of the symmetric skew-t equations for the tail parameter γ and k for our model. The expressions of the second derivative of the log density for observation from (2).

$$\ell(\gamma, k) = \sum_{t=1}^T \left\{ \ln C_t(u_{t1}, u_{t2}, \dots, u_{tn}; k_t) + \sum_{i=1}^n \ln f_{i,t}(x_{it}; \gamma_i) \right\} \quad (26)$$

$$\begin{aligned} \ell(\gamma, k) = & \ln \Gamma\left(\frac{\gamma+n}{2}\right) - \ln \Gamma\left(\frac{\gamma}{2}\right) - \frac{n}{2} \ln(\gamma\pi) - \frac{1}{2} \ln(\Sigma_t) \\ & - \frac{\gamma+n}{2} \ln \left(1 + \frac{x'_t \Sigma_t^{-1} x_t}{\gamma} \right) \\ & + \ln \left(1 + k \frac{(x'_t I_n)^2}{x'_t \Sigma_t^{-1} x_t + 1} \right) \quad (27) \end{aligned}$$

First order and second order derivatives of (27) w.r.t γ and k are,

$$\frac{\partial \ell_t}{\partial \gamma} = \frac{1}{2} \Gamma\left(\frac{\gamma+n}{2}\right) - \frac{1}{2} \Gamma\left(\frac{\gamma}{2}\right) - \frac{n}{2\gamma} - \frac{1}{2} \ln \left(1 + \frac{x'_t \Sigma_t^{-1} x_t}{\gamma} \right) + \frac{\gamma+n}{2} \left(\frac{x'_t \Sigma_t^{-1} x_t}{\gamma(\gamma+x'_t \Sigma_t^{-1} x_t)} \right) = 0 \quad (28)$$

$$\frac{\partial^2 \ell_t}{\partial \gamma^2} = \frac{1}{4} \left(\Gamma^{(1)}\left(\frac{\gamma+n}{2}\right) - \Gamma^{(1)}\left(\frac{\gamma}{2}\right) \right) + \frac{n}{2\gamma^2} + \frac{x'_t \Sigma_t^{-1} x_t}{\gamma(\gamma+x'_t \Sigma_t^{-1} x_t)} - \frac{\gamma+n}{2} \left(\frac{x'_t \Sigma_t^{-1} x_t (2\gamma+x'_t \Sigma_t^{-1} x_t)}{\gamma^2(\gamma+x'_t \Sigma_t^{-1} x_t)^2} \right) \quad (29)$$

For k :

$$\frac{\partial \ell_t}{\partial k} = \frac{\frac{(x'_t I_n)^2}{x'_t \Sigma_t^{-1} x_t + 1}}{\left(1 + k \frac{(x'_t I_n)^2}{x'_t \Sigma_t^{-1} x_t + 1} \right)} = 0 \quad (30)$$

$$\frac{\partial^2 \ell_t}{\partial k^2} = - \frac{\left(\frac{(x'_t I_n)^2}{x'_t \Sigma_t^{-1} x_t + 1} \right)^2}{\left(1 + k \frac{(x'_t I_n)^2}{x'_t \Sigma_t^{-1} x_t + 1} \right)^2} \quad (31)$$

Since (28) and (30) are not in closed form and cannot be solved explicitly, an iterative procedure is required to solve them numerically. The asymptotic normalized symmetric skew-t model of γ and k is normal distribution, $\sqrt{n}(\hat{\gamma} - \gamma, \hat{k} - k)' \sim N_2[0, H^{-1}(\gamma, k)]$. Where $H(\gamma, k)$ is the Hessian matrix,

$$H = \begin{bmatrix} \left(\frac{\partial^2 \ell_t}{\partial \gamma^2} \right) & \left(\frac{\partial^2 \ell_t}{\partial \gamma \partial k} \right) \\ \left(\frac{\partial^2 \ell_t}{\partial k \partial \gamma} \right) & \left(\frac{\partial^2 \ell_t}{\partial k^2} \right) \end{bmatrix} \quad (32)$$

This would also help us compare distributions to use Akaike and Bayesian information criteria. The corrected dynamic conditional correlation specification for dynamic dependence is estimated by using the inference functions for marginals (IFM). For each marginal i , estimate copula parameters by maximizing the marginal log likelihood:

$$\hat{\phi}_i = \arg \max_{\phi_i} \sum_{t=1}^T \left\{ \ln C_t(u_{t1}, u_{t2}, \dots, u_{tn}; k_t) + \sum_{i=1}^n \ln f_{i,t}(x_{it}; \gamma_i) \right\} \quad (33)$$

From these fits, compute standardized residuals $\hat{z}_{it}$ and for given $\hat{u}_{it}$, estimate copula parameters $\hat{\phi}_i$ by maximizing the copula log-likelihood:

$$\hat{u}_{it} = F_{skt}(\hat{z}_{it}; \hat{\phi}_i) = \frac{1}{T+1} \text{rank}(\hat{z}_{it}) \quad (34)$$

$$\hat{\phi}_i = \arg \max_{\phi_i} \sum_{t=1}^T \ln C_t(\hat{u}_{1t}, \hat{u}_{2t}, \dots, \hat{u}_{nt}; \hat{\phi}_i) \quad (35)$$

IFM is computationally convenient because we avoid joint maximization over a large ϕ and C .

2.6. Theoretical properties

2.6.1. Asymptotic properties

The asymptotic variance of $\hat{\phi}_i$ must account for estimation error in margins. Let $\hat{\phi}_i$ denotes stacked marginal estimates. Define the copula log-likelihood score w.r.t ϕ_i .

$$s_t(\phi) = \nabla_{\phi} \ln C_t(u_t; \phi) \quad (36)$$

And let $I_{\phi} = -E[\nabla_{\phi} s_t(\phi)]$, IFM gives the

$$\sqrt{T}(\hat{\phi} - \phi_0) \rightarrow N(0, V_{\phi}), \text{ where } V_{\phi} = I_{\phi}^{-1} [E[s_t s_t'] + B a^{-1} B'] I_{\phi}^{-1'} \quad (37)$$

Here equation (36) and (37) capture the sensitivity of the copula score to the margin parameters from equation (13). I_{ϕ} is the Fisher information for the marginal parameters and B is the cross-derivative matrix:

$$B = E[\nabla_{\phi} \ln C_t(u_t; \phi) * \nabla_{\phi} \ln f_t(x_t; \phi)'] \quad (38)$$

Practically computing the variance is complex: A robust approach is: compute standard errors by bootstrap (block bootstrap to preserve time dependence) or numerical Hessian of the copula log-likelihood at $\hat{\phi}$ ignoring margins. IFM is efficient relative to fully parametric MLE only when marginals are correctly specified. Otherwise IFM still produces consistent but possibly less efficient estimates. In finite samples IFM is much more practical.

2.6.2. Dynamic copula parameters

We will maximize the log-likelihood plugging in the method $\phi_t(\omega)$ governed by static hyper-parameter ω . By using equation (26) in IFM of u_t from ARMA-GJR-GARCH-skew-t marginals. ϕ_t is the time varying and evolves through a low-dimensional parameter vector K_t via some link $\phi_t = \Lambda(K_t)$. So, our choices are a linear correlation matrix (LCM) R_t , a vector of pairwise correlations and additional copula parameters such as degrees of freedom γ . We will build K_t with generalized autoregressive score GAS(1,1) and dynamic conditional correlation-type. The derived equations are Gaussian copula, Student-t copula, GAS(1,1) on a probability distributions proxy Q_t recursions.

$$\nabla_R \ell_t = \frac{1}{2} (R_t^{-1} y_t y'_t R_t^{-1'} - R_t^{-1}), \quad \nabla_s \ell_t = \frac{1}{2} (R_t - y_t y'_t) \quad (39)$$

$$\nabla_R \ell_t = \frac{1}{2} \left(R_t^{-1} \left(\frac{\gamma+n}{\gamma} \right) q_t q'_t R_t^{-1'} - R_t^{-1} \right), \quad \delta_t = q'_t R'_t q_t \quad (40)$$

$$Q_t = \bar{Q} + (\sqrt{a}I)(y_t - Iy_{t-1} - R_{t-1})(\sqrt{a}I)' + (\sqrt{b}I)Q_{t-1}(\sqrt{b}I)' \quad (41)$$

The closure form of the ARCH-GJR-GARCH-skew-t marginals are

$$\text{ARCH}(1): \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 \quad (42)$$

$$\text{GARCH}(1,1): \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (43)$$

$$\text{GJR-GARCH}(1,1): \gamma \varepsilon_{t-1}^2 I\{\varepsilon_{t-1} < 0\} \quad (44)$$

In this way, dynamic copula models provide a unified and theoretically grounded bridge between marginal volatility

dynamics and time-varying systemic risk, enabling more accurate measurement of diversification benefits, contagion effects, and downside protection in risk-sensitive portfolio construction.

3. Data and preliminary analysis

3.1. Data description

The dataset used in this paper is obtained the official website of the National Stock Exchange of India (NSE)

<https://www.nseindia.com>. We ensure that the analysis captures the temporal variations and patterns in this market datasets.

3.2. Summary statistics

We apply two-step estimation procedure called the IFM method from we find out the summary of our real datasets.

Table 1: Summary Statistics of four assets returns

Tools	GovtBond	CorporateBond	Equity	Real estate trusty's
Mean	0.053569	0.048886	0.050567	0.059724
Standard Deviations	0.048293	0.068494	0.240385	0.186731
Skewness	0.020256	-0.054137	-0.298708	-0.099260
Kurtosis	0.002054	0.629979	3.586132	2.459043
P-value of JB test	0.759292	0.000010	0.000010	0.000010
P-value of Engle test	0.414285	0.000010	0.000010	0.000010

The correlation matrix of four assets.

$$LCM = \begin{pmatrix} 1.0000 & 0.0046 & 0.0060 & -0.0012 \\ 0.0046 & 1.0000 & 0.0009 & 0.0006 \\ 0.0060 & 0.0009 & 1.0000 & 0.7776 \\ -0.0012 & 0.0006 & 0.7776 & 1.0000 \end{pmatrix}$$

The Table 1 presents the descriptive statistics of the returns of four asset classes, which include the mean, standard deviation, skewness and kurtosis, as well as the Jarque–Bera normality test and Engle’s heteroscedasticity test. Mean and standard deviation values are annualized. Skewness and kurtosis values are essential for risk management because they represent the fat-tailed characteristics of a return distribution.

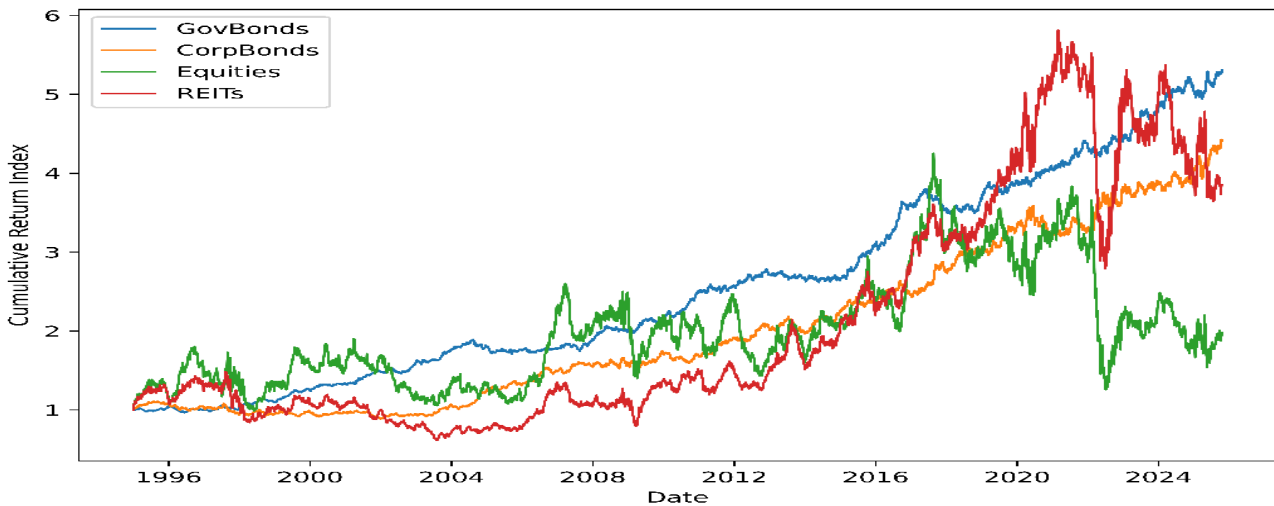


Figure 1: Cumulative Returns of Four Asset Classes (India, 1995–2025)

The Fig. 1 cumulative returns of four asset classes. This figure depicts the cumulative returns of Indian government bonds (GB), Indian corporate bonds (CB), Indian equities (EQ), and Indian real estate investment trusts (RT). The start point is February 1ST 1995 and end point is February 1ST 2025. The Jarque–Bera test

reject the normality at the 1% significance level, suggesting the necessity of using non-Gaussian distributions. In addition, Engle’s (1982) heteroscedasticity test also reject no conditional heteroscedasticity at the 1% significance level, indicating that capturing volatility clustering is crucial.

4. Empirical results

4.1. Marginal estimation

Table 2: Parameter estimates of marginal models.

Parameters	GB	CB	EQ	RT
Mean parameter				
μ (Const)	0.0208*** (0.0034)	0.0189*** (0.0047)	0.0324** (0.0145)	0.0293** (0.0117)
ρ (AR [1])	-0.0485 (0.0311)	0.7864*** (0.0384)	-0.2645*** (0.0462)	-0.0594*** (0.0465)
Variance parameter				
ω	0.7695*** 0.2683	0.2810*** (0.0727)	3.9721*** (0.7854)	2.2027*** (0.4213)
α	0.0244* (0.0140)	0.0262*** (0.0050)	0.0522*** (0.0071)	0.0411*** (0.0059)
γ	-0.0244 (0.0215)	-0.0019 (0.0058)	-0.0072 (0.0082)	0.0027 (0.0073)
β	0.1599 (0.2828)	0.9594*** (0.0066)	0.9311*** (0.0085)	0.9395*** (0.0070)
Residual distribution				
γ (df)	17.0630*** (3.7138)	4.4032*** (0.5886)	7.8878*** (1.5824)	6.5824*** (0.9971)
ℓ (skew-t)	-0.01000 (0.0161)	-0.0087 (0.0170)	-0.0498*** (0.0175)	-0.0195 (0.0169)
KS P-value				
KS P-value	0.9678	0.9318	0.8260	0.9529
Note: *** ($p < 0.01$), ** ($p < 0.05$), and * ($p < 0.10$) denote statistical significance at the 1%, 5%, and 10% levels, respectively				

The Table 2 reports the parameter estimates of marginal models for four asset classes (GB, CB, EQ and REIT). The unconditional means for all the asset classes are statistically significant. The mean is indicating that GB and CB exhibit low return characteristics whereas EQ and REIT show higher returns. The residual parameter is negative for GB, EQ and REIT, and positive

for the CB with statistical significance at 1% level of except for GB. We confirming base on variance GB and CB are low risk, and EQ and REIT are high risk. The degree of freedom parameter of γ is skew-t distribution is also statistically significant across all asset classes, confirming heavy-tailed behavior. GB indicating that most asset classes exhibit negative skewness in their return distributions.

4.2. Copula comparison

Table 3: Parameters estimate for all dynamic copula models

Parameters	DNC	DtC	DGHSktC	DSCSktC
AIC	-1384	-1363	-1386	-1398
BIC	-1165	-1134	-1189	-1142

The Table 3 give estimates of the parameters of various versions of dynamic copula models. The reveal the dependence structure between the asset classes. Among the four models, DSCSktC has the lowest AIC (-1398), which indicates an optimal fit, although it

has relatively higher BIC than the other models. In general, the findings show that there are a great time-varying dependence and persistence through assets classes, and equity and real estate markets have stronger connections.

4.3. Significance of symmetric dependence structure in portfolio management

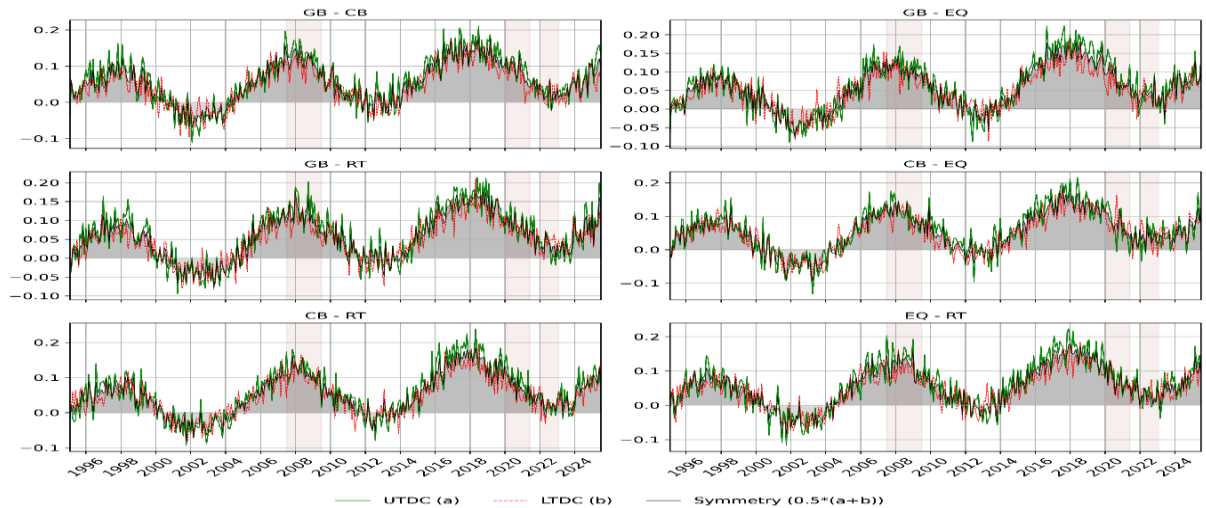


Figure 2: Tail dependence coefficient and symmetry based on DCSktC

The Fig. 2 considers the economic implications of capturing the symmetric dependence structure among multi-asset classes. We compare the four dynamic copulas by comparing the accuracy of ES estimation and the performance of the minimum ES portfolio within out-of-sample analysis. Each multivariate model consists of four types of dynamic copula and the name marginal models. We first describe the back-testing procedures and then discuss the results. Figure 2 shows the time series of UTDC (green color solid line) and LTDC (red color dotted line) between Indian government bonds (GB), Indian corporate bonds (CB), Indian equities (EQ), and Indian REITs (RT). The time series of symmetry ($\lambda U + \lambda L$) is

also displayed in same graph on the both sides. In addition, multiple periods of financial market turbulence points, including the 2007-2008 global financial crisis, the COVID-19 crash on 2020 and the Russia-Ukraine war 2022 high inflation period, are show in each plot with red shaded.

4.4. Tail risk and portfolio implications

Tail risk and portfolio implications, denote the study of extreme co-movements across asset classes of risk under the factor of market stress and the most effective portfolio diversification.

Table 4: Lower and upper quantile dependence of dynamic symmetric copula models.

Quantile	Pair	DNC	DtC	DGHSktC	DSCSktC
1%	GB, CB	0.0209	0.1849	0.1884	0.2031
	GB, EQ	0.1434	0.0441	0.1153	0.0743
	GB, RT	0.1581	0.1484	0.1095	0.0633
	CB, EQ	0.0698	0.0918	0.1932	0.1839
	CB, RT	0.1620	0.1197	0.0651	0.1998
	EQ, RT	0.1417	0.1366	0.2019	0.1537
5%	GB, CB	0.1075	0.0651	0.2436	0.2128
	GB, EQ	0.1703	0.2000	0.1656	0.2436
	GB, RT	0.0342	0.0672	0.0495	0.1183
	CB, EQ	0.0822	0.0816	0.2140	0.1249
	CB, RT	0.0649	0.1331	0.0696	0.2185
	EQ, RT	0.0319	0.2175	0.2022	0.0917
95%	GB, CB	0.0930	0.1792	0.0819	0.1580
	GB, EQ	0.1148	0.0388	0.1676	0.0858
	GB, RT	0.0304	0.2103	0.2428	0.2198

	CB, EQ	0.0687	0.0486	0.1837	0.1424
	CB, RT	0.0395	0.1241	0.0472	0.2410
	EQ, RT	0.0614	0.1559	0.1055	0.1592
99%	GB, CB	0.0799	0.2106	0.1937	0.1757
	GB, EQ	0.0450	0.0596	0.0522	0.2319
	GB, RT	0.1162	0.1645	0.0443	0.2537
	CB, EQ	0.1532	0.0703	0.0782	0.0885
	CB, RT	0.0687	0.1297	0.1307	0.1112
	EQ, RT	0.1179	0.0565	0.1014	0.1269

The Table 4, gives estimates of the quantile dependence between each dynamic copula model. Strong lower tail dependence is found at the 1% especially between GB and CB (up to 0.2031) and between CB and EQ (0.1932), which implies having more joint downside risk when markets are in extreme conditions. In the same way, the dependence of EQ -RT also has significant dependence (0.2019), which indicates that the risky assets are subject to contagion. Dependence tends to increase at the 5 percent quantile, particularly when using DGHSktC and DSCSktC models and is more pronounced at average stress levels. Dependence is still high in upper tail (95% and 99%), where GB, RT (0.2428, 0.2537) and CB, RT (up to 0.2410) demonstrate co-movements, which suggests the possibility of joint gains. DGHSktC and DSCSktC also have a high capture of tail dependence across models and this is due to

their capability to model asymmetry and heavy tails. All in all, it can be seen that the findings indicate that in extreme market temperatures there is limited diversification benefit, especially between equity and real estate investment and that bonds are a better hedge against certain circumstances.

4.5. Expected shortfall modelling

Expected Shortfall (ES, or Conditional Value at risk) is a popular metric of risk, which describes the average value of losses up to or above a specific risk amount. In contrast to the Value at Risk (VaR), which is merely a value that points to the largest loss to be faced within a certain confidence level, expected shortfall offers data on the magnitude of the extreme losses in the tail of the distribution.

Table 5: Precision of expected shortfall forecasts

Conf level(1-q)	Equal-weight portfolio				Risk parity portfolio			
	DNC	DtC	DGHSktC	DSCSktC	DNC	DtC	DGHSktC	DSCSktC
1%	0.0414	0.0407	0.0335	0.0313	0.0312	0.0319	0.0273	0.0246
5%	0.0539	0.0517	0.0463	0.0407	0.0405	0.0393	0.0345	0.0341
95%	0.0577	0.0568	0.0505	0.0450	0.0446	0.0453	0.0365	0.0365
99%	0.0625	0.0641	0.0546	0.0484	0.0549	0.0545	0.0471	0.0452

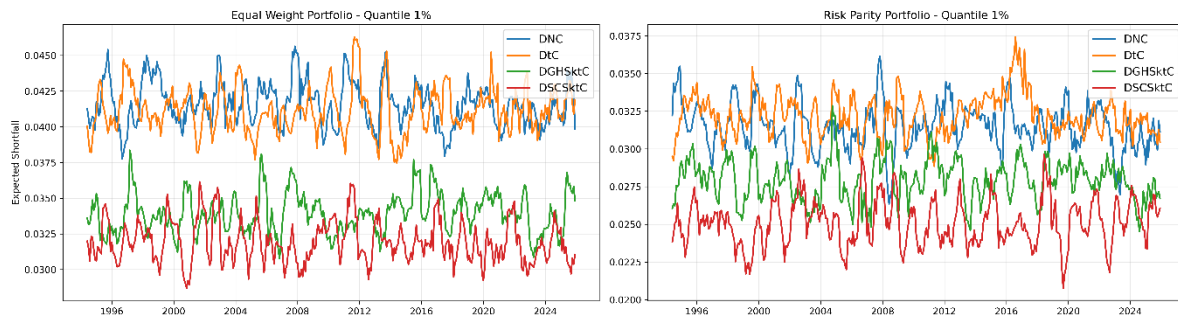




Figure 3: Precision of expected shortfall forecasts through equal weight and risk parity portfolio

The Fig. 3 demonstrate that DGHSktC and DSCSktC models are outperform DNC and DtC across all quantiles under equal-weight and risk parity portfolios by occurring lower Expected Shortfall. As a lower Expected Shortfall results in better downside risk mitigation. These findings checked that DGHSktC and DSCSktC

models are perform better in downside risk management. Among the models, DSCSktC has the lowest level of risk. Followed by DGHSktC. Alternatively, DNC and DtC are associated with a higher risk. Considering skewness and dynamic dependence leads to better risk control and more robust performance.

Table 6: Precision of expected shortfall forecasts of first and second half period

Conf level	Equal-weight portfolio				Risk parity portfolio			
(1-q)	DNC	DtC	DGHSktC	DSCSktC	DNC	DtC	DGHSktC	DSCSktC
First half period								
1%	0.5312	0.5174	0.4931	0.4765	0.6289	0.6031	0.5857	0.5711
5%	0.8675	0.8421	0.7965	0.7543	0.9622	0.9347	0.8989	0.8654
95%	0.9258	0.9015	0.8469	0.8123	1.0123	0.9746	0.9281	0.8924
99%	1.2048	1.1662	1.1159	1.0714	1.3087	1.2791	1.2313	1.1924
Second half period								
1%	0.5028	0.4873	0.4721	0.4632	0.5884	0.5673	0.5504	0.5382
5%	0.8293	0.8111	0.7756	0.7392	0.9408	0.9132	0.8787	0.8511
95%	0.9064	0.8758	0.8437	0.8121	0.9894	0.9583	0.9264	0.8928
99%	1.1547	1.1286	1.0943	1.0579	0.9894	1.2238	1.1862	1.1581

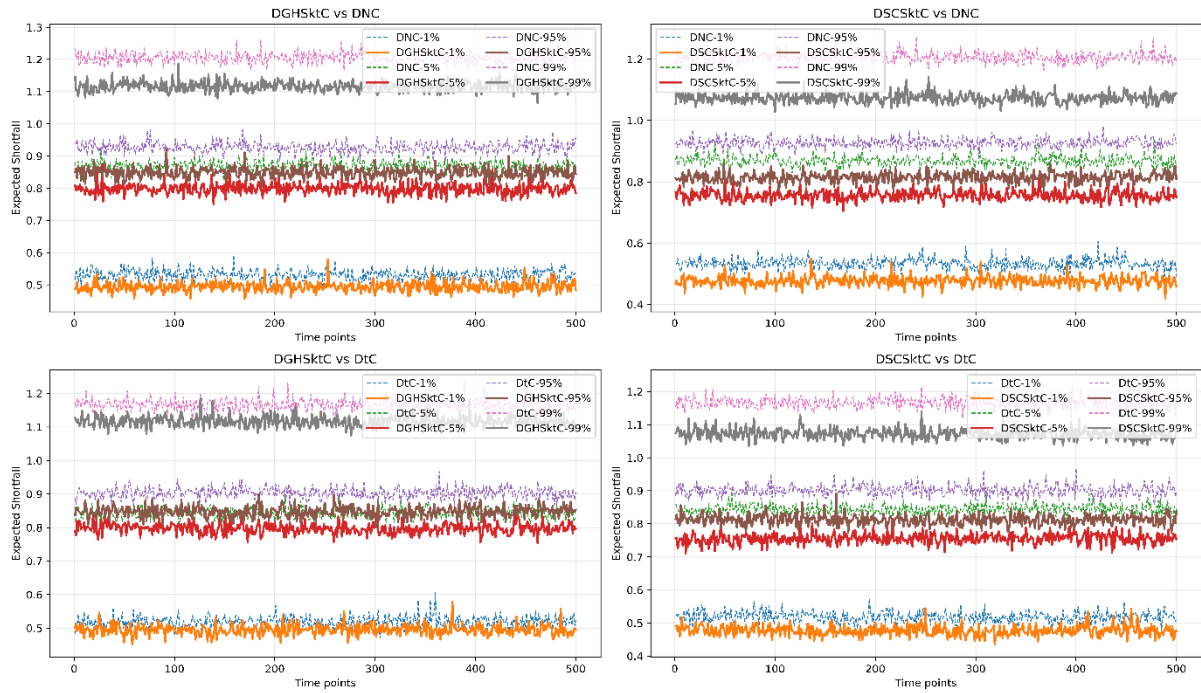


Figure 4: Precision of expected shortfall forecasts of first and second half period

In the Fig. 4, the four-panel plot illustrates a comparative evaluation of Expected Shortfall (ES) forecasts for the proposed DGHSktC and DSCSktC models versus traditional benchmark DNC and DtC models at tail probabilities of 1%, 5%, 95% and 99%. A closer examination reveals that forecast trajectories of models DGHSktC and DSCSktC are generally more robust, smooth and tightly aligned to their respective quantile levels than the ones of DNC and DtC. This suggests improved predictive accuracy and estimation stability over the 500 time points. The upper-left and lower-left panels show that DGHSktC shows reduced variability and better fit with DNC and DtC, especially in the higher (99%) and lower (1%) tails. The forecast time series displays smoother behaviors, implying better performance during

the simulated increase in volatility. Similarly, in the upper-right and lower-right panels, we see that DSCSktC exhibits even less fluctuations and variability, particularly in the left (1% and 5%) tail, which is essential for assessing downside risk. Both new derived models have smaller fluctuations over time than their counterparts, DNC and DtC, suggesting greater efficiency in tail dependence and dynamic risk modelling. This suggests that, of the two proposed methods, DSCSktC may be slightly better with narrower forecast ranges and less temporal variability. In summary, the findings provide strong evidence in favor of DGHSktC and DSCSktC as superior models for forecasting ES; they are more accurate, consistent and sensitive to tail risks, compared with traditional competing models, based on the timepoints.

4.6. Out-of-sample performance

Table 7: Performance of the minimum expected shortfall portfolio.

Conf level	Metric	Minimum Expected shortfall				Equal-weight
$(1 - q)$		DNC	DtC	DGHSktC	DSCSktC	
1%	Mean	0.0635	0.0652	0.0674	0.0691	0.0565
	Downside deviation	0.0733	0.0721	0.0716	0.0709	0.1452
	Sortino ratio	0.8662	0.9045	0.9421	0.9745	0.3897
	Maximum drawdown	0.2514	0.2472	0.2423	0.2389	0.4236
	Calmar ratio	0.2541	0.2632	0.2795	0.2911	0.1336
	Treynor ratio	0.0635	0.0652	0.0674	0.0691	—
	Information ratio	0.6245	0.7923	0.8559	0.8994	—
5%	Mean	0.0628	0.0660	0.0685	0.0692	0.0565
	Downside deviation	0.0728	0.0714	0.0708	0.0702	0.1452

	Sortino ratio	0.8624	0.9233	0.9728	0.9823	0.3897
	Maximum drawdown	0.2499	0.2402	0.2321	0.2328	0.4236
	Calmar ratio	0.2518	0.2742	0.2943	0.2961	0.1336
	Treynor ratio	0.0628	0.0660	0.0685	0.0692	—
	Information ratio	0.7032	0.8127	0.9053	0.9476	—
95%	Mean	0.0626	0.0661	0.0680	0.0686	0.0565
	Downside deviation	0.0726	0.0712	0.0707	0.0701	0.1452
	Sortino ratio	0.8628	0.9317	0.9712	0.9748	0.3897
	Maximum drawdown	0.2498	0.2394	0.2308	0.2309	0.4236
	Calmar ratio	0.2506	0.2768	0.2962	0.2957	0.1336
	Treynor ratio	0.0626	0.0661	0.0680	0.0686	—
	Information ratio	0.6844	0.7733	0.8627	0.9031	—
99%	Mean	0.0622	0.0666	0.0675	0.0672	0.0565
	Downside deviation	0.0722	0.0708	0.0704	0.0699	0.1452
	Sortino ratio	0.8593	0.9442	0.9595	0.9661	0.3897
	Maximum drawdown	0.2542	0.2341	0.2253	0.2259	0.4236
	Calmar ratio	0.2443	0.2835	0.2992	0.2974	0.1336
	Treynor ratio	0.0622	0.0666	0.0675	0.0672	—
	Information ratio	0.6021	0.7456	0.8184	0.8662	—

The Table 7 presents portfolio performance under the Minimum Expected Shortfall (MES) for four different probability levels (1%, 5%, 95% and 99%) of DNC, DtC, DGHSktC and DSCSktC models vs Equal-weight benchmark. Overall, these findings clearly show that the proposed DGHSktC and DSCSktC models have better risk-adjusted performance than the traditional DNC and DtC models. First, the Mean returns are the highest for DGHSktC and DSCSktC for all confidence levels. At 5% confidence level for example, the mean return increases from 0.0628 (DNC) and 0.0660 (DtC) to 0.0685 (DGHSktC) and 0.0692 (DSCSktC), reflecting a superior profitability. Second, the two proposed models show lower downside deviation, indicating improved downside risk

management. At 99%, downside deviation decreases from 0.0722 (DNC) and 0.0708 (DtC) to 0.0704 (DGHSktC) and 0.0699 (DSCSktC). Third, the Sortino ratio (return / downside risk) is largest for the proposed models. At 1%, it improves from 0.8662 (DNC) and 0.9045 (DtC) to 0.9421 (DGHSktC) and 0.9745 (DSCSktC). Likewise, Maximum Drawdown is lower for DGHSktC and DSCSktC, showing lesser maximum potential losses, and Calmar ratios are also higher, signaling better returns to drawdown ratios. The higher Treynor and Information ratios for DGHSktC and DSCSktC show better systematic risk-adjusted returns and higher active management efficiency.

4.7. Portfolio optimization results

Table 8: Average portfolio weights in duration of external factor effects (Global inflation, Covid-19 and Russia-Ukraine war)

Assets	GB	CB	EQ	RT
DNC	0.310	0.660	0.020	0.010
DtC	0.295	0.682	0.015	0.008
DGHSktC	0.288	0.701	0.010	0.008
DSCSktC	0.292	0.693	0.011	0.004
Maximum drawdown	0.278	0.132	0.164	0.247

The [Table 8](#) shows the asset allocations weights under various types of copula models and the drawdowns maximum. The weight of the corporate bonds (CB) in all models has the highest weight (ranging between 0.660 and 0.701), which implies that it has the most significant contribution to risk-minimizing portfolios compared to other models, which have moderate weightings (around 0.288–0.310) in GB. By comparison, the weights of equity (EQ) and real estate (RT) typically have lower weights (usually lower than 0.02) because they contribute more risk to the optimal portfolios and are less desirable than other assets. The allocation moves a little more towards CB and less exposure to EQ and RT (improved tail-risk sensitivity) with more advanced copula models (DGHSktC and DSCSkC). Maximum drawdowns indicate that CB has the least drawdown (0.132), which helps to justify its stability, and GB (0.278) and RT (0.247) have comparatively higher drawdowns. The findings show that bond-heavy levels of allocation, especially in CB increase the stability of a portfolio and

reduce the downside risk within DGHSktC and DSCSkC copula structures.

4.8. Economic interpretation

The [Panel A](#) shows cumulative returns 2008–2009. It is clear that the DtC model performs better in the recovery phase as it has a strong and persistent upward movement and has the highest cumulative value (around 3.0). DNC, on the other hand, exhibits a relatively poor growth, only slightly less than the starting point and ends slightly higher than 1.3. DGHSktC and DSCSkC show moderate yet consistent changes, which end at around 1.6, with DSCSkC slightly more volatile. Generally, DtC is the most beneficiary of good market conditions whereas DNC is the least beneficiary. The cumulative returns over 2020–2022, as shown in [panel B](#), are also defined by an increased volatility. In this case, DGHSktC proves to be the most successful model, showing steady growth, and terminating at around 1.5. DNC is followed by slow improvement, showing the flexibility to the changing conditions.

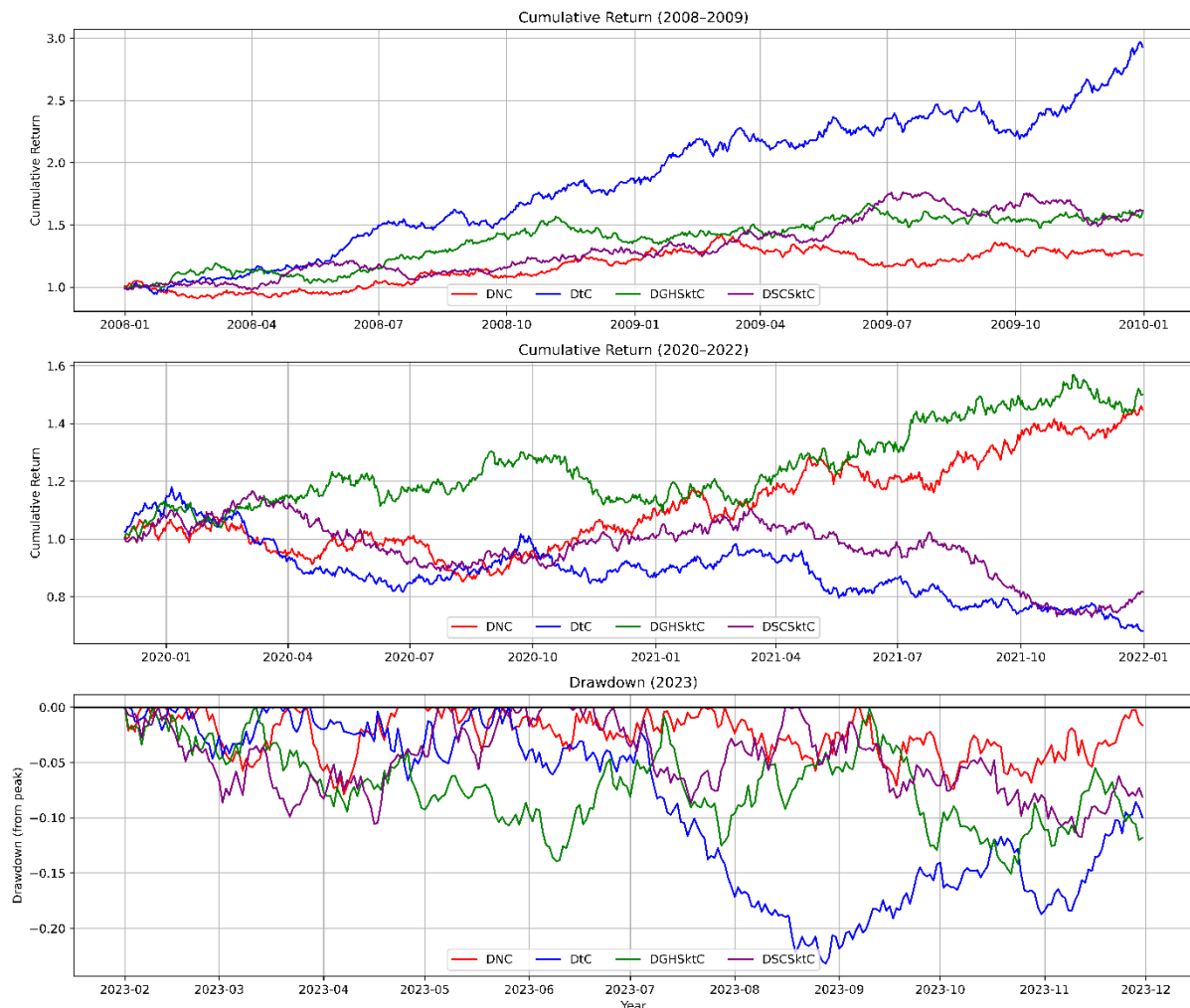


Figure 4: The downfall of the catastrophic events between 1995 and 2025

DSCSkC demonstrates performance that is both mixed with significant variability but no significant trend. DtC on the other hand has a poor performance with a continuous downward trend and concludes way lower than it started. This reversal implies that DtC is vulnerable to unstable market regimes, but DGHSktC and DNC are more resilient. The drawdowns for 2023 are shown in

[panel C](#), with emphasis on risk characteristics. The minimal drawdowns of DNC are generally within the range of -5 to -8, which indicates good risk management. DSCSkC exhibits moderate drawdowns, which means that the risk is balanced. DtC suffers greater losses, in the range of -20, implying greater volatility. The worst drawdowns are recorded in DGHSktC, falling

as far down as -70% which implies that there is a significant amount of downside risk even though it has had the earlier benefits of returning. Comprehensively, whereas DGHSktC has higher returns in some periods, DNC has most stable and risk-managed performance in various market conditions.

5. Discussions

5.1. Interpretation of results

The empirical evidence supports the relevance of modeling time-varying symmetric heavy-tail dependence in a multi-asset portfolio. The marginal analysis confirms the fact that government bonds (GB) and corporate bonds (CB) are characterized with low-risk and low-return characteristics, whereas equities (EQ) and REITs (RT) have higher returns and higher volatility profiles. The results of the copula indicate that dependence structures are not fixed, but rather very dynamic with higher correlations being recorded between risky assets (EQ-RT and CB-EQ) especially when the market is under stress. The persistence parameters near to unity depict dependence long memory, which supports the necessity of dynamic model. Notably, superior models like the DGHSktC and DSCSktC excel over the traditional copulas as shown by higher AIC values and improved tail dependence fit. Tail dependence and Expected Shortfall (ES) analysis results indicate that extreme co-movements are heightened in times of crisis, decreasing the benefits of diversification. Additionally, the DSCSktC model continues to give better ES estimates and better portfolio performance, which validates the importance of the inclusion of symmetric tail dependence in financial modeling.

5.2. Financial implications

The implications of the findings on the portfolio management and risk control are of importance. First, there are indications that conventional diversification solutions might not work when there are extreme market situations, such as heightened tail-dependence, especially in equity and real estate assets. Thus, dynamic copula-based risk models should be integrated by investors in order to capture extreme co-movements. Second, the higher performance of minimum-ES portfolios, particularly with DSCSktC and DGHSktC models, demonstrates that tail-risk-based optimization works better than the DNC and DtC models such in equal-weight portfolios. Risk parity approaches also promote more stability by decreasing ES and time-smoothing risk. Furthermore, the results of the portfolio allocation indicate that there is a high preference of corporate bonds (CB) as they are stable and less experiencing downside due to their low drawdown properties whereas exposure to equities and REITs are kept to a minimum because they have greater downside risk. Lastly, the time-varying ES dynamics show that risk is very sensitive to economic cycles and as such, the traditional risk management frameworks cannot be relied upon. Investors are advised to use adaptive and forward-looking strategies that consider the variation in dependence structures and tail risks.

5.3. Policy relevance

Policy wise, the findings highlight the importance of healthy systemic risk monitoring frameworks in emerging economies such as India. Strong tail dependence in crises also means that there is a greater chance of financial contagion between asset classes and this may increase systemic instability. The dependence measures and ES metrics based on copulas should thus be included by regulators

like central banks and financial authorities in their risk surveillance mechanisms. Moreover, the fact that the diversification benefits are decreasing at times of the COVID-19 crisis and inflationary shocks indicates that the market interconnectedness is growing that needs more active macroprudential policies. Multi-asset tail risk scenarios should be part of stress testing frameworks in order to test financial systems vulnerabilities in a more efficient way. Finally, the results justify the use of ES as a desirable risk measure compared to VaR and are consistent with the international regulations, including Basel III and IV. The policies can enhance financial stability, capital adequacy test, and preparedness to crisis by incorporating new-fangled dependence models and tail-risk metrics in dynamic market settings.

6. Conclusion

This paper developed and empirically validated a dynamic symmetric skew-t copula framework to capture time-varying heavy-tail dependence among Indian multi-asset classes. The results demonstrate that incorporating both symmetric and skewed tail dynamics significantly improves the accuracy ES estimation and enhances portfolio performance during periods of financial turbulence. The proposed DSCSktC and DGHSktC models consistently outperform conventional symmetric copulas (DNC, DtC) in terms of lower tail-risk exposure, higher risk-adjusted returns, and better resilience across crisis regimes such as the 2008 Global Financial Crisis, the 2020 COVID-19 crash, and the 2022 R-U war inflationary period. Portfolio back-testing confirms that DSCSktC achieves the most stable and conservative allocation, minimizing drawdowns and improving capital preservation, while DGHSktC excels in capturing extreme downside co-movements at ultra-low quantiles. These findings emphasize that dynamic copula modeling of symmetric heavy-tail dependence provides a more robust framework for risk management and diversification in emerging markets. This paper contributes to the growing literature on copula-based portfolio construction by offering empirical evidence that quantile-specific copula selection can substantially strengthen financial stability and enhance the effectiveness of minimum-ES strategies in dynamic market environments.

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Author Contributions: All authors contributed to the conceptualization, methodology development, data analysis, and manuscript preparation, and have approved the final version of the paper.

Data availability: The daily return data of Indian government bonds (GB), corporate bonds (CB), equities (EQ), and REITs (RT) from 1995 to 2025, was compiled by the authors using data retrieved from publicly available financial sources (including RBI, NSE, and SEBI archives). The processed dataset used in this analysis is available upon reasonable request from the corresponding author.

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